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NOTES ON THE PRESSURE OF RADIATION

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AND

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ABSTRACT. § 1 consists of a brief account of some early experiments on radiation pressure, dealing in particular with the investigations of Lebedew and of Nichols and Hull. In § 2 Dr Hull reconsiders some aspects of his work, in view of the errors of calculation pointed out in these *Proceedings* by Mary Bell and S. E. Green. In § 3 the latter authors reply to Dr Hull's note.

§ 1. A RÉSUMÉ OF SOME EARLY EXPERIMENTS ON RADIATION PRESSURE

BY S. E. GREEN, PH.D.

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Introduction. The phenomenon of radiation pressure is of considerable importance in modern science, more particularly in astrophysics, where it constitutes a prominent factor to be taken into account by astronomers in speculations concerning the state of heavenly bodies. The search for the existence of the phenomenon in the case of terrestrial sources of light has an interesting history.

Although experiments successful in detecting the pressure of radiation have been confined to the present century, this pressure has been the subject of research since comparatively early times. The idea that light could exert a pressure seems first to have been advanced by Kepler*, who in 1619 thereby explained the repulsion of comets' tails by the radiation of the Sun. In the middle of the eighteenth century De Mairan and Dufay† conducted laboratory tests. The emission theory of light being then generally accepted, a pressure due to radiation incident upon a receiving surface was to be expected from the arrest of the flying corpuscles. The results obtained were, however, quite inconclusive. The subject was subsequently

* De Mairan, *Traité Physique et Historique de l'Aurore Boréale*, p. 357 (2nd edition. Paris, 1754).
† De Mairan, *loc. cit.* p. 371.

taken up by a number of investigators including Fresnel in 1825, Crookes, commencing in 1874, and Zöllner at about the same time. Fresnel* was merely able to show that the forces he observed were not of electric or magnetic origin, while in the other cases no definite detection of the pressure of radiation resulted. This is not surprising in view of the extreme smallness of the effect sought for†, together with the relatively enormous gas action which was generally produced, owing to the heating of the body receiving the radiation. Crookes's‡ interest in the subject led to the invention of his well-known radiometer, while Zöllner§, who used somewhat similar apparatus, calculated that under certain circumstances gas effects could give rise to forces many thousands of times greater than that of the radiation pressure to be expected.

Maxwell‖ in 1873 had given a new impetus to the search by his deduction that if light be an electromagnetic wave phenomenon, there exists in a beam a pressure which is exerted in the direction of travel and is of magnitude numerically equal to the energy per unit volume of the beam. It followed that the total pressure exerted on a normally reflecting surface should be equal to the sum of the energy-densities of the incident and reflected beams. A few years later, Bartoli¶ established independently from thermodynamical reasoning that radiation should exert such a pressure, but failed, in common with all previous observers, in the attempt to detect and measure the pressure, owing to gas action in his apparatus.

The definite detection of radiation pressure, under conditions in which extraneous gas effects are fairly small and not dominant, has been achieved only during the last thirty years or so. During this period a few attempts have been made to test the validity of Maxwell's relation directly by separate measurement of the pressure and energy-density of radiation. The common method adopted in the case of the pressure has been to direct a stream of radiation normally upon a delicately suspended vane forming the arm of a torsion balance. Then, if gas action be absent,

P the pressure P due to the incident beam follows from the relation $(1 + m)\, PrA = c\theta$,

m, A where m is the reflecting power of the vane, A the area of the spot of light on the

r, c vane, r the arm at which the pressure acts and c the torsional constant of the suspension. The energy-density of the radiation has generally been found by the calorimetric method of observing the rise of temperature produced by the incidence of the beam upon a body of known thermal capacity.

The main gas effects which may occur are convection and radiometer action. The usual practice has been to evacuate the containing vessel as far as possible in order to reduce gas action to a minimum. In the experiments described below Lebedew sought to eliminate this action entirely by suitable construction of the vanes, while Nichols and Hull, whose investigation was unique in the use of an unexhausted chamber, attempted to get rid of gas effects by a ballistic method.

* *Ann. de Chim. et de Phys.* **29**, 57, 107 (1825).

† In the case of a beam of sunlight the pressure amounts to less than 10^{-4} dyne/cm^2.

‡ *Phil. Trans.* **164**, 501 (1874); **169**, 243 (1878); **170**, 87 (1879).

§ *Pogg. Ann.* **160**, 154, 296, 459 (1877).

‖ *Treatise on Electricity and Magnetism* (Oxford, 1873).

¶ *Sopra i movementi prodotti della luce e dal calorie* (Florence, Le Monnier, 1876); Exner, *Rep. d. Phys.* **21**, 198 (1884).

Lebedew's experiments. Lebedew's work, published at the beginning of this century* is of great importance as being the first investigation to give positive results. The torsion system having a glass fibre was mounted within a flask 20 cm. in diameter. A number of vane systems were used of which that shown in figure 1 is an example. The arm r was about 1 cm. long. The vane materials adopted were platinum, aluminium and nickel, with blackened surfaces in some cases. The source of radiation was an arc lamp O, the light from which could be directed upon either surface of any vane within the flask K by means of the optical system, duplicated in part, shown in figure 2. Variations in the intensity of the source were corrected for by reflecting a portion of the beam on to a thermocouple system L. The response of the galvanometer in circuit with the latter was recorded simultaneously with the deflections of the suspension.

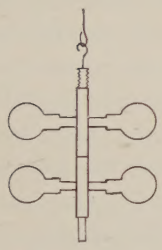

Figure 1.

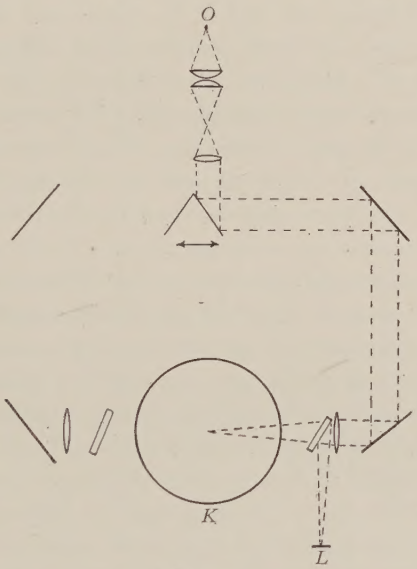

Figure 2.

Lebedew attempted to evacuate the flask down to the vapour pressure of mercury at the temperature of a freezing-mixture of ice and salt, so that the residual gas pressure was probably of the order of 0·0001 mm. He argued that convection effects would be small at such a pressure, and could be nullified by making the vanes as vertical as possible and letting the radiation fall upon each side in turn. Further the attempt was made to eliminate radiometer action by using vanes of different thicknesses and deducing the deflections for infinitely thin vanes. Lebedew does not give many deflectional data in his account, but it appears that the angular rotations of the suspension, read by means of a mirror on the stem and a scale some distance from the flask were about 0·4°. Owing to the vacuum, the suspension was in continual oscillation, the mean position being derived from the end points of the swings. The torsional constant of the suspension was found from the change in

* *Ann. d. Phys.* (4) **6**, 433 (1901).

period produced by the addition of a body of known inertia. The normal period was about 5 sec. The reflecting powers of the vane-surfaces were measured by means of a photometer.

For determinations of the energy-density of the beam, the whole projection system was moved along a slide so that the radiation could be directed upon one face of a copper block, the rise in temperature of which was registered by a mercury thermometer inserted through the upper face. The water equivalent of the block was about 3 gm. A piece of glass was placed in front of the calorimeter to compensate for the presence of the side of the flask during the pressure-observations. In the later experiments the copper block was enclosed within a constant-temperature bath. From simultaneous readings of the deflections of the galvanometer in circuit with the thermocouple system L, figure 2, the energy-density was deduced for the standard beam, for which the value was about 25×10^{-6} erg/cm^3. The standard incident beam for both pressure- and energy-measurements was that which corresponded to a deflection of 100 divisions on the galvanometer scale.

Lebedew admits the possibility of errors of 20 per cent in the determinations. Owing to the great diminution of the intensity of the radiation, observations which were taken with coloured light are less reliable than those taken with full radiation. Even in the latter case the final results for the ratio of pressure to energy-density appear to have varied from 0·6 to 1·4. Much of the inconsistency probably arose from uneliminated gas action. Later observers have found rather large radiometer effects to occur at about the gas pressure used by Lebedew, but in the specimen table of deflections given it appears, from the constancy of the turning points, that in that particular case gas action was not very prominent. The marked degree of success which Lebedew achieved may in large measure be attributed to his use of thin metal vanes. Considering the small deflections of the suspension, and the comparatively rough method of determining the energy-density of the radiation, precise measurements were not to be anticipated. Although Lebedew's results are only to be regarded as qualitative, his work constituted a very great advance beyond any previous to it, in giving positive results of the right order.

The experiments of Nichols and Hull. The best known investigation on the subject is that of Nichols and Hull*, who used great ingenuity and experimental skill in an attempt to obtain a quantitative test of Maxwell's relation. For vanes they used cover glasses silvered on one side. These were supported at the base of the suspension, which was mounted within a bell jar 11 cm. in diameter, figure 3. The suspending fibre was of quartz. A rough plan of the apparatus is shown in figure 4. Radiation from the arc lamp O was brought to a primary focus by a lens system S_1, and to a secondary focus, in the plane of the suspension, by the lens S_2. By placing the latter against the stops p_1 and p_2 in turn, the light could be directed on to either of the vanes as required. The deflections of the suspension were observed by means of a telescope and scale T_2 and a small mirror i on the suspension. To obtain deflections with the radiation incident upon both the glass and the silvered surfaces of each vane, the suspension was rotated through 180° by turning the control magnet

* *Proc. Amer. Acad.* **38**, 559 (1903); *Phys. Rev.* **17**, 26, 91 (1903).

M outside the bell jar, a magnet g, figure 3, being mounted on the upper part of the suspension for this purpose. The same means was used to turn the vanes out of the way when the arm, about 0·8 cm. long, at which the radiation operated was being measured, this measurement being made by observing, through the travelling telescope T_1, the two positions of the spot of light produced in the plane of the suspension. The torsional constant of the balance was computed from its period and its calculated moment of inertia.

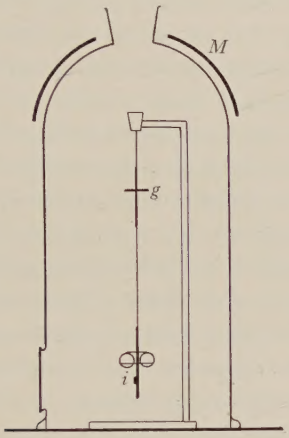

Figure 3.

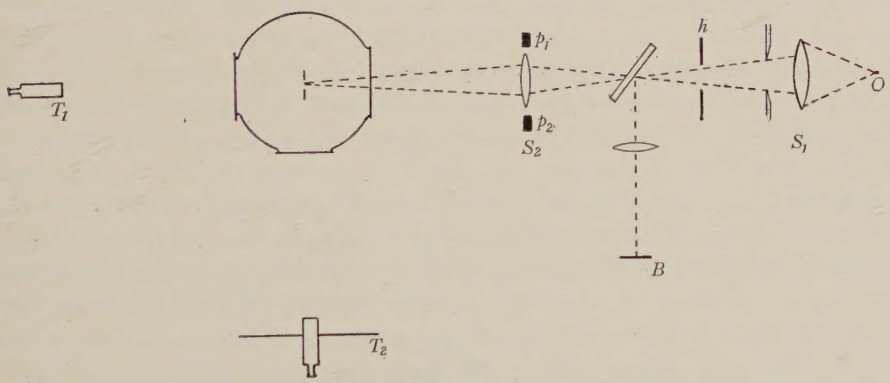

Figure 4.

A remarkable feature of Nichols and Hull's experiments was the relatively high gas pressure employed in the torsion chamber. Taking as a guide the fact that, whereas radiation pressure acts instantaneously, gas action takes time to develop, Nichols and Hull concluded, from a preliminary series of tests in which radiation was directed upon the vanes for various intervals of time, that gas action was at a minimum in their torsion chamber at a pressure of about 16 mm. of mercury. The bulk of the main experiments were then conducted at this pressure. Under these circumstances the damping of the motion of the suspension was large and care had to be taken to make the vanes vertical, otherwise the balance was very sensitive to

convection currents. Further, Nichols and Hull directed the radiation upon each vane, not permanently, but for a few seconds only. They argued that any gas action which might develop in the former case would not have time to do so in the latter. The shutter h was operated by a magnetic escapement governed by the seconds contact of a standard clock, so that the radiation could be allowed to fall upon the vane for any integral number of seconds. The selected time of exposure was six seconds, one quarter of the balance period. With the radiation incident for this interval only, the suspension being initially at rest, the first maximum deflection of the vane system was observed. The throw on the scale T_2, about one metre distant, was roughly 30 mm. The corresponding steady deflection, deduced by calculation, was about 16 mm. for the standard beam, referred to later.

The reflecting-powers of the vane-surfaces were determined by the use of a bolometer. During the pressure-measurements the silver surfaces deteriorated rapidly* and required frequent renewal. To obtain representative average values of the reflection coefficients, observations were made on a number of fresh silver coatings deposited on the same vanes. The mean reflecting power on the silvered sides was 92 per cent and on the glass sides 78 per cent. Thus the reflecting powers of the surfaces actually used in the pressure experiments were never measured.

The energy-density of the radiation was found by removing the bell jar and directing the beam upon the blackened surface of a silver disc of known thermal capacity. The rate of temperature-rise of the disc was measured as it passed through the temperature of its surroundings. For this purpose two insulated thermocouples were inserted into the disc, which was enclosed within a water jacket provided with a suitable orifice to admit the radiation. The couple leads, suspending the disc, passed upwards through a flat wooden box into a lagged can containing kerosene in which the constant-temperature junctions were immersed. The thermocouple systems were connected in series, together with a 1000-ohm resistance and a galvanometer G_1. The circuit was calibrated by immersing the disc in baths having temperatures a few degrees above and below that of the laboratory. The opening to the disc chamber was covered by a piece of plate glass similar to that comprising the windows of the bell jar. The disc was cooled before being enclosed by the water jacket, and the radiation was then allowed to fall upon it. The energy-density of the beam was deduced from the subsequent rate of change of deflection of the galvanometer G_1. The slight imperfection in the absorptive power of the disc surface was allowed for, but lag of the galvanometer and of the thermocouples seem to have been disregarded. It is uncertain, however, how far the claim to a final accuracy of $\frac{1}{2}$ per cent was justifiable†, in view of the use of mercury thermometers to measure all the necessary temperatures and of the neglect of lag effects, which to some extent must occur in all variable-state methods.

The circuitous experimental methods made the evaluation of the final results very involved. In a manner similar to that adopted by Lebedew, Nichols and Hull

* Nichols and Hull state that during their preliminary experiments the silvered surfaces rarely lasted for more than two evenings' work.

† Apart from the error in the mechanical equivalent of heat, referred to later on page 596.

attempted to correct for fluctuations in the intensity of the source of light by reflecting a fraction of the radiation by means of a glass plate on to a suitable receiver. The latter was the bolometer B, figure 4, in circuit with which was a second galvanometer G_2, the deflections of which were recorded during both the pressure and the energy-density observations. All results were then reduced to the values relating to a constant permanently incident beam of standard intensity, the beam being chosen so as to produce a steady deflection of 100 divisions on the scale of galvanometer G_2.

It is to be noticed that during the pressure-measurements, owing to the short exposures used, Nichols and Hull observed not steady deflections but ballistic throws of both the vane system and the galvanometer G_2. From the throw of the torsion balance was derived the displacement α at which the couples due to the radiation and the elasticity of the fibre would be equal. This required the division of the observed throw by a calculated factor F of about 1·3. Then, by means of an experimentally determined correction amounting to 50 per cent, the ballistic throw of the galvanometer G_2 was converted to the steady deflection δ which would be produced by the same beam if incident permanently. The assumption was made that the radiation pressure was proportional to the intensity, and the deflection of the torsion balance corresponding to the standard incident beam was then $100\alpha/\delta$. Nichols and Hull did not perform their calculation in this order, but the above description affords a simple way of explaining the nature of the reduction which they carried out.

An additional complication arose from the fact that the sensitivities of the galvanometers which were both of the moving magnet type, varied and needed to be checked continually. One cause of disturbance was the rotation of the control magnet M outside the bell jar. Nichols and Hull had therefore to reduce all their values of pressure and of energy-density to correspond to standard sensitivities of the galvanometers*, before deriving results relating to the standard incident beam.

The ultimate comparison took the following form. The total radiation pressure due to the incident and the reflected components of the beam was compared with $(1 + m)$ times the energy-density E of the standard incident beam. The final figures given by Nichols and Hull were as shown in table 1.

Table 1

Total pressure (10^{-5} dyne/cm^2)	$1{\cdot}85E$ (10^{-5} erg/cm^3)	Difference (per cent)
7·01 (± 0·02)	7·05 (± 0·03)	− 0·6

In addition to observations for the full radiation from the arc lamp, two further sets of readings were taken with the same radiation after it had passed through ruby glass and through a water cell. The final figures for the latter two cases are not quite

* Although not definitely stated to be so, in the account, it appeared that these galvanometer sensitivities were defined in opposite ways.

so important as those quoted above, owing to the diminution in the intensity of the light and the correspondingly smaller observed deflections. However, the calculated results are raised to relate to the same standard incident intensity as before and show agreements between the pressure and the energy-density within + 1·1 and − 0·6 per cent respectively.

The above published figures have gained general acceptance as quantitatively confirming, to their stated accuracy, the validity of Maxwell's relation; and for a generation they have been widely quoted as conclusive.

The publication of these figures had, however, two disadvantages. In the first place these figures are not the true results of Nichols and Hull's experiments. It was noticed not only that a wrong value of the mechanical equivalent of heat had been used in the computation of the energy-densities, but also that errors of calculation occurred in the evaluation of the pressures. An erroneous value was taken for the conversion factor F used to reduce the ballistic throws of the vane system to static conditions*. Correct evaluation of Nichols and Hull's results shows the pressure to exceed the energy-density by from 9 to 11 per cent of the latter, the values for full radiation giving the best agreement.

The second disadvantageous effect of the published figures was the creation of a false impression as to the precision attainable, without very great difficulty, in a comparison experiment of this kind. Not only are the pressure and the energy-density, in the case of any laboratory source of light, exceedingly small, but there are so many quantities to be measured and so many corrections to be applied, in addition to possible errors of optical setting, that it is obviously very difficult to obtain final values of pressure and energy-density accurate to less than a few per cent, even if the source be steady and gas action absent. Considering the indirectness of Nichols and Hull's methods, and the small deflections of their torsion balance, the agreement shown in their results, even when correctly calculated, was very satisfactory.

The agreement shown by Nichols and Hull's true results was not definitely improved upon for 30 years. Poynting and Barlow in 1910†, working with blackened and polished surfaces, obtained agreement up to 12 per cent between pressure and energy values, but could not entirely get rid of gas action. Fraulein Golsen‡ showed, in 1924, that radiometer action could be eliminated at a sufficiently high vacuum with metal vanes, and derived agreement varying from + 6 to − 4 per cent. She, however, admits errors of 6 per cent in the measurements. It does not seem that any of the above workers was aware of Nichols and Hull's errors. In the paper by Mary Bell and S. E. Green already referred to, results were given for pressure and energy values which appear to show somewhat better agreement than any hitherto obtained.

* These errors are discussed by Mary Bell and S. E. Green, *Proc. Phys. Soc.* **45**, 320 (1933).
† Poynting, *Collected Works*, p. 381 (Cambridge, 1920); *Proc. R. S.* A, **83**, 534 (1910).
‡ *Ann. d. Phys.* **73**, 624 (1924).

§ 2. A HISTORICAL STATEMENT

By G. F. Hull, Ph.D.

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In the March, 1933, number of the *Proceedings of the Physical Society*, Mary Bell and S. E. Green call attention to errors of calculation in the paper on "The Pressure of Radiation" by E. F. Nichols and G. F. Hull, *Proc. Am. Acad.* **38**, 559 (1903). One of these errors, that of the use of the wrong value for the mechanical equivalent of heat, was detected by the writer two or three years after the publication of the article, and in various addresses, notably that before the American Physical Society later referred to, the correct value was given. But the substitution of the correct value did not greatly alter the result, for in the published report the pressure through air as determined by the light-balance was less than that computed from the energy-density by 0·6 per cent. Substituting the correct value made it too large by 1·4 per cent. The discrepancies for red glass and water cell were about 3 and 1·4 per cent, but as the deflections in the three cases were of the order of 100, 60 and 30 chief importance attaches to the through-air readings. The red glass was slightly uneven in quality and variations in its readings might have been due to this fact.

Concerning the factor by which the ballistic throws were reduced to static readings the following considerations may be set forth. First it appeared that the optimum time of exposure would be nearly one quarter of the period. In our case the actual time had to be a whole number of seconds. We chose 6 sec. as this time. It was called a quarter of the period by courtesy. The differential equation was set up and a solution which dispensed with quantities amounting to a fraction of one per cent was obtained.

For example, the equation $\frac{\partial^2\theta}{\partial t^2} + 2\gamma\frac{\partial\theta}{\partial t} + G\theta = L$ has for its solution* θ, t, γ, G

$$\theta = \frac{L}{G}\left[1 - e^{-\gamma t}\sqrt{(1+x^2)}\cos\left(2\pi\frac{t}{T} - \delta\right)\right],$$

given that $\theta = \partial\theta/\partial t = 0$ when $t = 0$. Here $e^{-\gamma\frac{T}{2}} \cong 0{\cdot}80$ and $x = rT/2\pi = \tan\delta \cong 0{\cdot}070$. x, T
Hence $\sqrt{(1+x^2)} \cong 1{\cdot}0024 \cong 1$.

When $\sqrt{(1+x^2)}$ is replaced by 1, δ also drops out of the equation for values of t near zero or $T/2$. But it does not drop out when $t \cong T/4$, a fact which was not evident when we worked out the solution. However, the most serious error was in inadvertently dropping out of the computation the factor $e^{-\gamma t}$ where γ is of the order of 0·02 and t of the order of 2·7 sec., though this factor appears in the solution

* If the light be cut off at time t_1 then thereafter t_1

$$\theta = Ae^{-\gamma t}\cos(2\pi t/T - \alpha),$$

where $A^2 = (L/G)^2[(1+x^2)(1+e^{-2\gamma t_1}) + 2e^{-\gamma t_1}\sqrt{(1+x^2)}\{x\sin(2\pi t_1/T - \delta) - \cos(2\pi t_1/T - \delta)\}]$ A

and $$\tan\alpha = \frac{x + e^{-\gamma t_1}\sqrt{(1+x^2)}\sin(2\pi t_1/T - \delta)}{1 - e^{-\gamma t_1}\sqrt{(1+x^2)}\cos(2\pi t_1/T - \delta)}.$$ α

Then $$\theta\,(\max) = \frac{Ae^{-\gamma t_2}}{\sqrt{(1+x^2)}},\ \text{where } t_2 = T(\alpha - \delta)/2\pi.$$ t_2

of the final equation. As a mathematical performance our solution for the general case was very faulty. It was carried out at first for the case in which $\gamma = 0$, in which it was correct. For that case the reduction factor is 1·414. It was then carried out for the first case in hand, in which $e^{-\gamma T/2} = 0{\cdot}83$ and $T = 23{\cdot}0$ sec. The exposure time was 6 sec. For that case it is very nearly true that $\theta\ (t = 6) = L/G$ and the reduction factor is 1·328. The faulty reduction factor 1·357 was worked out very early in the investigation and was used throughout, the thought being that variations in it due to damping or period could be allowed for later.

Through the long series of readings, whenever a ballistic deflection was taken and recorded a static deflection was taken. That is, light was allowed to fall upon the vane until at least twelve turning-points were read. Hence we always had data for determining the value of the pressure without recourse to the ballistic readings. But our light-source was an unsteady arc lamp; the light-balance and the reference galvanometer had different periods and damping coefficients. On the whole we relied on our ballistic readings, since the exposure time and therefore the chance of lamp-variation was twenty-four times as great in the static as in the ballistic measurement. Moreover the gas action in the ballistic method was less.

But our static readings, as also our ballistic, showed that for certain gas pressures radiometric action, as compared with light pressure, had been eliminated to the order of a few per cent.

In their 1900–1901 experiment* Dr Nichols and the writer used a bolometer for measuring the energy of the light-beam. In the data presented in our report we included all the results for air pressures from 96 mm. to 0·06 mm. This was done merely to show that over that large range the ballistic throws were fairly constant and nearly equal to the value demanded by theory. But choosing the region in which the gas action was obviously small, from 37·9 to 33·4 mm., we obtained reduced throws of 21·6, 22·1, 21·1, averaging 21·6 and, applying the correction due to the error in the reducing factor, we obtained the corrected value 23·1 mm.† The pressure corresponding to this deflection is $23{\cdot}1 \times 4{\cdot}65 \times 10^{-6}$ or $1{\cdot}075 \times 10^{-4}$ dyne/cm^2. When we reported our results in our first paper we had not measured the reflection coefficients for air-silver and glass-silver. We assumed these to be the same and nearly equal to 0·92. But on application of the correct reflection coefficients 0·92 and 0·77 the pressure should be equal to $\frac{1}{2}(1{\cdot}92 + 1{\cdot}77)E$ or $1{\cdot}85E$.

The energy-density as given in our first report was in error owing to the fact that the resistance of the bolometer as measured, 0·278 Ω., was too great on account of the silver having been eaten off the leading-in strips‡. A computed value 0·221 Ω. of the resistance was given in our second report. Another computed value, not given there but later worked out, was 0·227 Ω. This was obtained by subtracting from the measured resistance 0·278 Ω. the resistances of the small cylinder-sector strips, each 0·8 mm. long, 5·0 mm. wide, of the thin platinum, for which $\sigma = 0{\cdot}160$. If the mean resistance is taken as 0·224 Ω., the pressure of the standard beam computed from

* *Phys. Rev.* **13**, 307 (1901).

† For the above pressure-range the period was 24 sec. and the damping ratio 0·790; the correct reducing factor was 1·265.

‡ *Phys. Rev.* **17**, 32, 33 (1903).

energy-density should be $1{\cdot}85 \times 0{\cdot}224 \times 0{\cdot}750 \times 10^{7}$ or $1{\cdot}030 \times 10^{-4}$ dyne/cm^2. After allowance for the fact that the black coating diffusely* reflected 4·5 per cent of the incident radiation, the pressure as computed from the energy-density of the incident beam was $1{\cdot}075 \times 10^{-4}$ dyne/cm^2. The agreement between the two values of the pressure must be regarded as favoured by chance, since the resistance of the bolometer is so critically dependent upon that of the leading-in strips.

When it was stated in our second report that the average error was 6 per cent, the reference was to the results of all the readings for pressures from 96 to 0·06 mm. But when attention is confined to the region given, the average error for the 60 observations (20 for each pressure) was small. There might of course be a large constant error, but inspection of the data shows that the throws were rather free from gas action.

Coming to the data presented in our second and third papers†, there are the following points to be set forth. It was only towards the end of our work that we discovered by our measurements that the reflection coefficient for glass-silver was considerably less than that for air-silver, 0·77 as compared with 0·92. After subtraction of the 4 per cent reflected from the glass surface the absorption on the glass-silver surface was 27 as compared with 8 on the air-silver surfaces. If no gas action whatever was present the light pressures on the two surfaces should have been as 1·77 to 1·92. Since the two deflections were not in that ratio, gas action must have been present.

From table 3 we should then have the relations:

$$1{\cdot}92L + G = 17{\cdot}11,$$
$$1{\cdot}776L - 3{\cdot}3G = 16{\cdot}71,$$

where L is the pressure due to incident beam and G the gas action for air-silver surface; this assumes that the gas action is reversed when the vane is reversed.

It results that $1{\cdot}848L = 16{\cdot}67$ in place of 16·91, as given in table 3.

Since the period in nine of the observations was nearly 23·75 we find that the correct ratio of ballistic to static is 1·275. After allowance for the 0·8-per-cent correction already applied, the computed pressure is $16{\cdot}67 \times 1{\cdot}55/1{\cdot}265 \times 0{\cdot}363 \times 10^{-5}$ or $7{\cdot}40 \times 10^{-5}$ dyne/cm^2. The pressure from the corrected energy-density is $7{\cdot}05 \times 0{\cdot}98 \times 10^{-5}$ or $6{\cdot}91 \times 10^{-5}$ dyne/cm^2. The discrepancy therefore lies between 6·6 and 7 per cent. The discovery of the error in the ballistic reduction factor has, therefore, brought into agreement the values which were thought to differ by some few per cent, and caused those which were thought to be in close agreement to differ by 6 or 7 per cent.

Lebedew, whose paper on light pressure appeared about the same time as our first paper, rejected, as small, quantities less than 10 per cent and had variations between the two values of light pressure as high as 50 per cent for white light and 80 per cent for blue light. Yet his work is quoted at times as having verified the Maxwell-Bartoli relation.

It might now be enlightening to note the large errors or discrepancies in the

* *Phys. Rev.* **17**, 100 (1903). † *Phys. Rev.* **17**, 27, 91, (1903).

measurement of other quantities involving radiation, such as the Stefan-Boltzmann constant σ in the relation $E = \sigma T^4$. The *International Critical Tables* **5**, 273 give a list of the most important measurements. These range from 6·5 to 5·3 during the years 1909–1912. Of twenty-one measurements since 1909 only four can claim to have an accuracy of 1 per cent, that by Coblentz who, with all the excellent equipment of the Bureau of Standards at his command and after ten years of continuous labour on this kind of problem, led the way in 1915, and three others of very recent date. To these measurements should be added that of Hoare, whose excellent work was done in 1932. When it is remembered that light-sources and measuring devices have been vastly improved during the past thirty years, it can be seen that the work of Nichols and Hull—involving on the one hand measurements in an absolutely new and perplexing field, the disentangling of light pressure and gas action, and on the other hand measurement of the energy-density of a light-beam in absolute values—established quantitatively the Maxwell-Bartoli relation.

Nichols and Hull made light pressure as capable of exact measurement as other quantities in radiation, and this at ordinary gas pressures. But the writer knew that some gas action was present in their measurements and extended the investigation*. When great care was exercised in making the vane vertical it was shown that the deflection when light fell upon an enclosed black surface was to that for an enclosed silver surface in the ratio 1 : 1·58 for air pressures from 30 mm. to 160 mm. This is the ratio required by the Maxwell-Bartoli relation, as determined by the known reflection coefficients†. Clearly there could be no gas action, even on the black surface, much greater than 1 per cent of the light pressure. Hence on a silvered surface the gas action must have been of the order of ½ per cent. But unfortunately not all the data were published. In Note-Book 10, Third Series, Nov. 29-Dec. 8, 1904, there is a complete series of readings on an air-silver and glass-silver surface, neither surface being enclosed. Here the Maxwell-Bartoli ratio ought to be 1·085. It was found that for air pressures from 20 mm. to 160 mm. the average ratio was 1·08. No gas action as great as 1 per cent could have been present. Consequently it was clear that the chief difficulty in testing the Maxwell-Bartoli relation was that of measuring in absolute units the energy-density of the beam.

Again, before the American Physical Society at the Hanover meeting in 1908‡ the writer demonstrated the effect of a beam of light on a vane of clear glass, on two such vanes, on an enclosed black and on a silvered vane, and showed that the deflections had the ratios 1·0 : 1·7 : 5·6 : 11·5, while the Maxwell-Bartoli relation gave ratios 1 : 1·85 : 5·86 : 11·2; this for an air pressure of about 30 mm. It is beyond doubt that all the American physicists present were convinced that there was no difficulty in getting deflections due to light pressure with practically no gas action present.

Conclusion. When all corrections had been applied, the experiments of Nichols and Hull in 1900–1901 showed that the light pressure computed from the energy-

* *Phys. Rev.* **20**, 293 (1922).

† The light source in this case was no longer a sputtering arc lamp but a Nernst filament at a definite voltage.

‡ *Science*, **28**, 254 (1906).

density as measured by a bolometer agreed with that determined by the light-balance within 1 per cent. Again, when all corrections have been applied in the later series of 1901–1902, in which a silver disc was used in the measurement of the energy-density, the agreement is within 7 per cent. In view of the very much larger variations in corresponding absolute measurements of radiation which prevailed at that time it can be stated that the experiments of Nichols and Hull verified the theoretical relation of Maxwell and of Bartoli. Further it can be stated that Hull in later work showed that the ratio of the deflections for a black and a silvered surface agreed with the theoretical ratio within 1 per cent; therefore the gas action on the silvered surface could not have been much greater than $\frac{1}{20}$ per cent. To establish an agreement between the two values of light pressure to this degree of accuracy would necessitate measuring the energy-density of a light-beam, with an accuracy which is rather beyond our ability.

§ 3. NOTES ON PROF. HULL'S STATEMENT

By Mary Bell A.R.C.S., B.Sc., and S. E. Green, Ph.D.

Received March 20, 1934.

In our consideration of Nichols and Hull's work we confined our attention to the main experiments, which so far as can be seen from the published papers, were conducted with much greater care and with more attention to experimental detail than any of the earlier ones, and the published results for which have hitherto been generally accepted as conclusive.

Since, however, Dr Hull refers to earlier experiments we will consider *seriatim* the points he has raised. Regarding errors in the constant F, Dr Hull refers to some early experiment in which the period T of the torsion balance was 23·0 sec. and the ratio r* of successive deflections was 0·83, but with these values we are not concerned as they are not the values of the constants given in the main experiments to which our criticism relates. Further, the calculation of a reduction factor from these values seems to us to be irrelevant, since, as far as can be ascertained, they were not used in any of his published work. Our criticism in connexion with the errors of calculation in the main experiments remains valid, because Nichols and Hull reduced all observations to correspond with a value of 24 sec. exactly for T, and throughout the experiments r was 0·783. In these circumstances the correction factor F required to reduce the vane deflections to static conditions is 1·260 and not F 1·357.

As regards the static readings we feel from our own experience that very little reliance can be placed on these at the gas pressures at which Dr Hull worked. He mentions, however, that he relied chiefly on the ballistic observations.

Dr Hull next recalculates the results of a preliminary experiment which was reported much more briefly than the main work. In this early investigation radiation pressure was determined by methods similar to those already described. The

* With the notation employed by Nichols and Hull.

energy-density was found by means of a bolometer. The original published results* were as follows in c.g.s. units:

Total pressure	$1{\cdot}92E$
$1{\cdot}05 \times 10^{-4}$	$1{\cdot}34 \times 10^{-4}$

After the publication of the paper in 1901 it was discovered that in dissolving the silver away from the platinum when the bolometer was being made the acid had eaten into the leading-in strips of the bolometer. A direct correction to the resistance of the bolometer was impossible, but a computed correction of the energy values gave the following results, published in 1903†:

Total pressure	$1{\cdot}92E$
1·05	1·05

Probable errors of about 6 per cent are admitted in these values.

In his present note Dr Hull has made five further corrections of which (i) and (ii) relate to the pressure and (iii), (iv) and (v) to the energy-density. (i) The results for the range of gas pressures 37·9 mm. to 33·4 mm. (at which gas action appeared to be a minimum) have now been selected, the complete range of pressures having originally been from 0·02 mm. to 96 mm. This reduces the mean deflection from 22·5 mm. to 21·6 mm., a difference of 5 per cent. (ii) Revaluating his correction factor F, and avoiding errors in his calculation pointed out by us, Dr Hull has now used the value 1·265 instead of 1·357. (iii) Instead of the original assumed value of 92 per cent for the mean reflection coefficient of the vanes, a new value 85 per cent, obtained by direct experiment on silvered surfaces two years later, has now been used. (iv) A further correction to the bolometer resistance has been introduced. (v) Allowance has been made for the light diffusively reflected from the blackened surface of the bolometer.

The new results after all these corrections have been applied are

Total pressure	$1{\cdot}85E$
1·075	1·075

Dr Hull states that the final agreement was fortuitous, and this is obviously so since, all other corrections apart, the new corrections to the bolometer resistance (admitted by Dr Hull to be a very critical factor) bring into agreement pressure and energy values which otherwise would differ by 20 per cent.

We may mention that since the value of r was not given in the published papers, we were not able to check the value of the correction factor F for the preliminary experiments, as we had done in the case of the main work.

With reference to the main experiments outlined in the first part of this paper,

* *Phys. Rev.* **13**, 318 (1901).

† *Phys. Rev.* **17**, 33 (1903). Only significant figures have here been retained.

Dr Hull, who originally regarded gas action as negligible, has now recalculated the results on the basis that some gas action was present. This correction seems to us to be rather unreliable for two reasons. Firstly the correction applied only amounts to 0·2 mm. in a deflection of 16 mm., whereas during the experiment the zero wandered by more than 1 mm. Certainly the zero was always checked, but to apply the correction at all appears to be unduly straining the accuracy of the observations. Secondly, if his assumptions regarding gas action were correct, the ratio of any pair of observed deflections should be greater than 1·92/1·77. Out of eleven pairs of observations given by Nichols and Hull, in one case the deflections are equal and in four others the ratio is actually less than unity. Thus in these cases the gas action was in the contrary direction to that which Dr Hull postulated from the final mean values. If there had been a definite gas action it should certainly have acted in the same direction in every case. We feel also, from our own experience, that at the relatively high gas pressure used in his torsion chamber it was not possible to obtain gas effects constant enough to be satisfactorily eliminated by calculation. However, we can only express a tentative opinion on this point since the arrangement of our apparatus was different from that of Nichols and Hull.

Incidentally, in the calculation of the final value of the pressure it appears to us that the figure in the denominator should be the correct factor 1·260 and not 1·265 as the deflection 16·67 relates to a balance period of 24 sec. exactly.

We are glad to have the ambiguity in Nichols and Hull's use of their galvanometers removed and to learn that our inference was correct, namely, that galvanometer-sensitivity had been defined directly in the case of G_2 and inversely in the case of G_1. The sensitiveness was taken for the former instrument as proportional to the deflection for a given voltage across the terminals, and for the latter as proportional to the voltage producing unit deflection.

The correction for gas action now introduced by Dr Hull, even if justified, does not greatly affect the divergence between the pressure and energy-density results, which we have shown in the case of full radiation to be 9 per cent of the latter.* The errors we pointed out were merely of calculation, and do not detract from the experimental ingenuity with which Nichols and Hull conducted their classical investigation. We do, however, claim for our own methods the great advantages of simplicity and directness, with the consequent possibility of avoiding the majority of the complicated corrections inherent to the work of Nichols and Hull. In particular we may mention that modern research had made available to us a steady source of light and a ready means of obtaining a high vacuum, and further had provided a simple and accurate method of determining the energy of the incident beam. Our work has shewn that the high-vacuum method with the use of metal vanes is capable of disentangling radiation pressure from gas action with much greater certainty than any other method. With the advantages enumerated we naturally were able to obtain somewhat better agreement between pressure and energy values than could have been obtained from the unavoidably circuitous methods adopted by Nichols and Hull.

* See p. 596.

538.65

THE ACTION OF ALTERNATING AND MOVING MAGNETIC FIELDS UPON PARTICLES OF MAGNETIC SUBSTANCES

By H. STAFFORD HATFIELD, Ph.D.

Communicated by Prof. E. N. da C. Andrade, March 7, 1934. Read June 1, 1934.

ABSTRACT. The paper offers an explanation of the translatory movement observed by Mr W. M. Mordey in magnetic particles subjected to a multi-phase alternating field.

Some years ago Mr W. M. Mordey* discovered some striking phenomena exhibited by magnetic particles when exposed to single and multi-phase magnetic alternating fields. He found that particles of various magnetic substances, such as magnetite, specular haematite, and pyrrhotite ($Fe_n S_{n+1}$), when strewn upon a surface situated over one pole of a vertical single-phase alternating bar magnet, were in part attracted to the region over the pole and in part repelled away from it, forming a circular halo concentric with the pole. He also found that if the single-phase magnet were replaced by a row of poles excited alternately by two-phase current, the particles travelled in a direction along the line joining the poles.

A multi-phase magnet of this type is equivalent to a procession of alternate north and south poles travelling with a speed depending upon the periodicity of the supply. The particles of magnetic material travel in a direction opposite to that in which the field is travelling, although Mr Mordey does not seem to have stated this fact. They travel, in fact, in the opposite direction to that in which the edge of a sheet of metal is dragged by the induced eddy currents, as in the ordinary house-service alternating-current meter, the armature of which is an aluminium disc acted upon by a two-phase magnet.

No investigation leading to a clear physical explanation of these phenomena seems to have been published. Mr Mordey suggested that hysteresis might be concerned, and even that some minerals might be diamagnetic to alternating current, though paramagnetic to direct fields. The experiments described in the present paper were carried out with a view to finding some more satisfactory explanation. The frequency of the alternating current used was 150 c. sec. A microphone transformer served as a single-phase bar magnet, and a permanent magnet was arranged to rotate about an axis parallel to, and midway between, its limbs, and so to provide a moving field. A four-pole two-phase magnet was made of stalloy stampings, one pair of windings being connected through a non-inductive resistance, and the other through a condenser, to the alternating supply. The magnetic powders employed

* W. M. Mordey, *Phys. Soc. Proc.* **40**, 338 (1928).

were iron filings, magnetite, pyrrhotite, tungsten steel filings, and $\frac{1}{16}$-in. steel bearing balls. Globules of magnetite were also made, by fusion with an oxy-gas flame.

Mr Mordey's phenomena were easily reproduced, the rotating magnet giving results similar to those obtained with the two-phase magnet. It was found that when either of them was below the surface on which the particles rested, as in Mordey's arrangement, motion took place in the opposite direction to that of the field; when the magnet was above the surface, travel took place *with* the field. If, however, the two-phase magnet was brought too near to the surface, the particles ceased to travel and were attracted to the poles.

When the magnetic matter, either in the form of lump or of powder, was attached by adhesive to one arm of a sensitive torsion balance, no anomalous re-

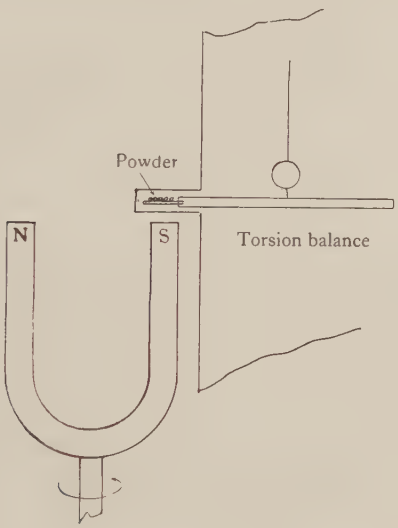

Figure 1.

pulsive or translatory force was observed in the field of the rotating permanent magnet, arranged as in figure 1, or in those of the single-phase and two-phase magnets. The only force observed was attraction in the direction of maximum increase of field-strength. If the arm of the torsion balance pointed towards the axis of rotation of the magnet, no deflection occurred. Yet loose particles resting upon a surface in the same position relatively to the magnet are driven, as if by a blast of air, in a direction opposite to the motion of the magnet poles. On the other hand, particles allowed to fall freely through the moving field are not deflected either in, or against, the direction of its motion. This indicates clearly that the action observed upon particles lying loose upon a surface is due to reaction between them and the surface.

A slip of mica coated with magnetic powder stuck on with an adhesive, and pivoted so as to turn in a horizontal plane about the same axis as the magnet, rotated in the same direction as the latter. This is the well-known hysteresis effect, commonly exhibited in the Ewing hysteresis tester. Mr Mordey refers to similar

arrangements in his paper, but they afford no explanation of the motion of free particles, which is in the opposite direction. Particles strewn upon a surface situated over the rotating magnet swirl round in the direction opposite to its rotation, being also drawn towards the axis, where they form a kind of whirlpool.

The single-phase magnet was found to repel the particles when below the surface, as in Mordey's arrangement, but to attract them when above it.

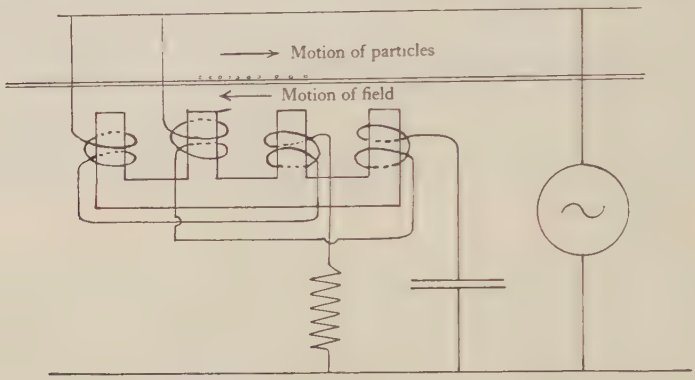

Figure 2.

The clue to an explanation was given by the fact that magnetite particles could be rendered almost inactive by exposing them to a strong alternating field for a little while, and restored to activity by exposing them to a unidirectional field. The first

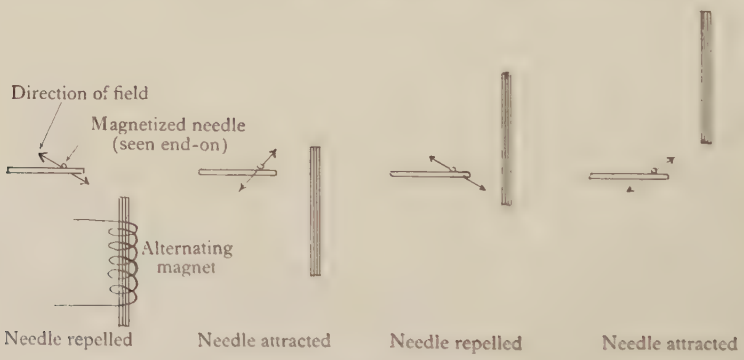

Figure 3.

process destroys their permanent magnetism, the second restores it. The mechanism of attraction and repulsion is easily demonstrated by means of the experiments shown in figures 3 and 4. A short piece of magnetized sewing-needle rests upon a horizontal glass plate, the length of the needle being at right angles to the line joining its centre to the axis of the bar magnet fed with alternating current. In figure 3 the needle is seen end-on.

We find that the needle moves towards or away from the magnet according to the level of the latter. If, as in figure 4, we arrange that one pole of a magnetized needle suspended by a thread shall rest upon the edge of a glass plate and if we act upon it

with the magnet in the various relative positions shown in figure 3, we find that the single pole now moves in the same sense as the fragment in figure 3.

The explanation which suggests itself is as follows. During one half of a cycle, the field is acting upon a pole with, say, an oblique downward force. The horizontal component of this force cannot move the particle along the surface, for it is opposed by friction under the combined weight of the particle and the vertical downward component of the magnetic force. During the other half of the cycle the vertical component tends to lift the pole; hence the horizontal component can act, and the pole moves. The fragment of needle resting with both its poles upon the glass progresses by stepping out, as it were, with each pole alternately. It owes its motion to the fact that either pole can only move when the force on it has a component in the upward direction.

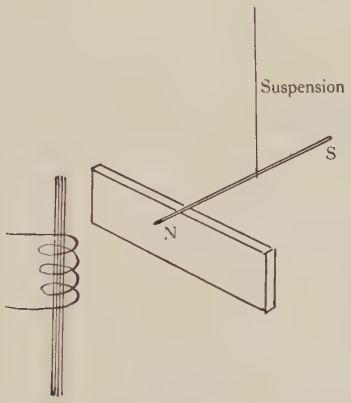

Figure 4.

The above explanation is confirmed by the fact that spherical particles, such as steel balls or globules of magnetite, are entirely inactive even when strongly magnetized. The poles are very close together, and the forces on them are almost exactly equal and opposite in any direction. The only effect they can have is to cause the sphere to oscillate slightly from side to side but not to progress, since it touches the surface only at one point.

The action of moving fields can be explained on similar lines. At any point past which a succession of field-magnet poles is travelling, figure 5, the direction of the field they produce at a given point undergoes complete revolutions; in the figure, the field poles are imagined as travelling to the right, and the field at any point on the surface thus rotates counter-clockwise. Its intensity in each direction is symmetrical on either side of the vertical. During one half of a cycle it possesses an upward or lifting component for one or other pole (N in the figure) of a particle resting on the surface; this component reaches a maximum half-way through the half-cycle. The direction of the horizontal component changes from urging the particle pole in the direction of travel of the field poles, through zero, to urging it in the opposite direction. But it can only act effectively after the lifting component has lifted the particle. Since the particle takes time to rise and fall, the horizontal component

urging it against the direction of travel of the field poles is effective for a longer time than the component in the opposite direction; the particle leaves the surface in the upward force at a point making a certain angle with the vertical, but does not return to the surface until the force makes a greater angle on the other side of the vertical. This is the clue to the phenomenon, as may be seen by studying the horizontal and vertical components of the force exerted on a north pole in different positions, as shown in figure 5.

It should be noted that, in this case, the revolution of the field must produce a torque on the particle, due to hysteresis, and tending to roll it along the surface in the observed direction. This, however, is very weak, for it is ineffective to produce motion in the very case where circumstances are most favourable to its action, namely that of spherical particles.

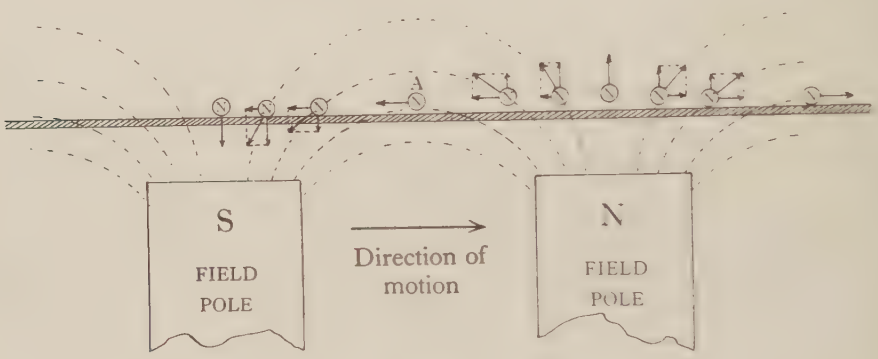

Figure 5.

As the phenomenon depends upon the particles being lifted by a field rapidly reversing in direction, it is clear that they must be small to respond to a high frequency. At 150 c. sec., particles of larger size than 60 mesh (say, 0·3 mm. in diameter) are very sluggish, and those over 20 mesh (say, 0·9 mm. in diameter) hardly move. There is no action, according to the above hypothesis, upon a particle the magnetic axis of which lies in the direction of the field; this is easily demonstrated with the magnetized needle. Hence the particles need to be small to be flung about by the field; they then travel when they happen to fall aright. The extremely lively motions of some of the particles are very striking; they strongly suggest gnats. If these lively particles are isolated they do not seem to differ in any way from the rest. Probably they are of such a size as easily to revolve synchronously with the field when in the air.

It is possible that a study of the demagnetization of magnetite by alternating current would have interesting results. It takes time of the order of seconds, and the material thus treated regains its activity spontaneously with time.

This work was carried out in the Physics Department of University College, London, under a Fellowship grant from the Leverhulme Trustees, to whom I beg to express my thanks, which are also due to Professor E. N. da C. Andrade for the kind and helpful interest which he has taken in the investigation.

NOTE ADDED JUNE 12, 1934

Since the above paper was read my attention has been drawn to a hypothesis put forward by Prof. J. A. Fleming in his book *Scientific Research and Electrical Engineering*, in which he attempts to explain the Mordey effect. He assumes that a particle of a ferromagnetic substance exhibiting hysteresis, when situated in the field of an alternating magnet may experience a repulsive force, owing, as he says, to the fact that "the induced pole in the particle nearest to the magnet will always be of similar polarity." Again: "The particles possess ferromagnetic permeability, in virtue of which they tend to move from weak to strong places in the magnetic field. But they also possess magnetic hysteresis, and in consequence there is a dissipation of energy. The particle, therefore, tends to move from strong to weak places in the field to make this dissipation a minimum, and its actual position depends upon the relative magnitude of the two forces."

It would appear that the repulsive forces considered by Prof. Fleming, if they exist, cannot possibly account for the major portion of the effect, since my torsion balance was amply sensitive enough to detect forces of the necessary magnitude. Furthermore, the fact that the Mordey effects are greatly increased by first exposing the particles to a unidirectional field, and so permanently magnetizing them, seems to contradict the assumptions made by Prof. Fleming.

DISCUSSION

Prof. S. CHAPMAN said that he had investigated Mr Mordey's proposed explanation of the phenomenon under discussion but had found that it yielded results of the wrong magnitude. The author was the first to hit upon the true explanation.

Mr S. N. RAY. The author is the first to have provided a physical explanation of the Mordey effect. This effect is being studied by me at the Sir John Cass Technical Institute under Dr D. Owen, but owing to the work being only part-time the progress has been slow. Specular haematite (Elba) and pyrrhotite (Norway) have been studied with an electromagnet having a stalloy core, and alternating current at 100 c./s. With a delicate torsion balance no anomalous diamagnetic repulsion has been observed either in lumps or with masses of powdered material. The author's explanation of the change of repulsion into attraction, as the level of the surface carrying the powder is lowered below that of the pole-faces, is a valuable contribution to our knowledge of this phenomenon.

Owing to the sluggishness and erratic movement of the particles with a rapidly alternating field it has been found convenient to use direct current, very slowly reversed with a commutator, for the electromagnet. With such a supply a microscopic study of the particles shows motion resembling a somersault or a forward jump rather than the stepping out of the fragment of needle described by the author. Again, the repulsion effect can still be produced when the active particles are enclosed in spheres of paraffin wax.

These two observations do not fit in with the author's account of the process but can be explained from the known magnetic properties of haematite*, namely, ferromagnetism with large hysteresis along the principal axis and different degrees of paramagnetism along the remaining axes. Such a crystal at rest in a unidirectional magnetic field will be subjected to a couple when the field is reversed, owing to the resultant magnetic intensity not being turned through 180° at the same time. It may well be, therefore, that Mr Mordey's opinion that the explanation is somehow bound up with the large hysteresis is not entirely without foundation.

It would be interesting to learn if the author has succeeded in producing the drift with particles of materials of high coercive force, such as cobalt steel or tungsten steel. If they are first magnetized to saturation by a unidirectional field at right angles to the alternating field and then subjected to the latter, the particles should step away from the pole as described by the author.

Author's reply. As I mentioned in the paper, I have observed the stepping-out motion in weak fields, but in strong ones the same forces are sufficient to cause the particles to be flung about quite violently.

I am not very clear as to Mr Ray's remarks on haematite. His observation that particles enclosed in wax spheres are repelled is interesting, and seems to contradict my observation that spherical particles will not move. It should be noted that the essential difference between the hysteresis theory and my own permanent-magnetism theory lies in the fact that in the first case the field is supposed to reverse the magnetization of the particles at each alternation, in the second not. The fact that particles lose their activity after long exposure to an alternating field and regain it after exposure to a unidirectional field seems to me decisive in favour of the second view.

As I explained in the paper, hard steel balls are the most favourable objects for the exercise of hysteresis effects, and yet neither they nor magnetite spheres are affected in the slightest. Cobalt-, tungsten-, and carbon-steel filings move vigorously.

The torsion balance excludes the theory given by Prof. Fleming. The particle needs the surface in order to move. Hysteresis could only act by rolling the particle along the surface; the tendency to roll is there but is too weak to be effective even in the most favourable case. We must conclude that the particle needs to retain a permanent polarity, which suffices to explain its motion.

* Townsend Smith, *Phys. Rev.* **8**, 721 (1916), **15**, 345 (1920).

535·412

THE APPLICATION OF THE THEORY OF THE TRANSMITTING ECHELON TO THE EXPLANATION OF TALBOT'S AND POWELL'S BANDS

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Received March 22, 1934. *Read June* 15, 1934.

ABSTRACT. On the basis of the theory of the transmitting echelon, the formation and the asymmetrical character of Talbot's bands, which are produced when a plate and aperture are placed in certain positions in the beam of a prism spectroscope, are explained analytically and represented diagrammatically. The effects of variation of aperture-width, plate-thickness and inclination of the echelon to the beam are considered. It is contended, in contradiction to A. W. Porter's conclusion, that perfect blackness of the dark bands is approached as the thickness of the plate increases. The appearance of bands when the plate and aperture are placed in front of the spectroscope slit is discussed. A direct diagrammatic representation is given of the formation of bands as photographed by A. C. G. Beach when prism and echelon dispersions are at right-angles. The fact, reported by N. K. Sethi, that in the case of Powell's bands the asymmetry can be reversed by suitable choice of liquid is accounted for, and the criterion for reversal is shown to be equivalent to that stated by Sethi.

§ 1. HISTORICAL REVIEW

The interference bands observed in a continuous spectrum produced by a prism spectroscope when a thin film of glass or mica is placed in one of three positions across a portion of the beam were first described by Talbot* in 1837. The most remarkable feature of these bands is that whereas they appear when the film is placed in the beam between the collimator and telescope, so that the intercepted part of the beam traverses the thicker part of the prism, no bands can be observed when the film is placed towards the base of the prism so that the unintercepted part of the beam traverses the thinner part of the prism. Again, the bands can be seen if the film intercepts the beam emerging from the eyepiece on the side towards the violet part of the spectrum but not if the other side of the beam is intercepted. The explanation of the bands as given by Talbot was incomplete in that this asymmetry in the method of production of the bands was not accounted for.

A similar system of bands was described by Powell† in 1848. Into a hollow glass spectroscope prism containing a highly refractive liquid is inserted a plate of glass with its edge parallel to the refracting edge of the prism to intercept a portion

* H. F. Talbot, *Phil. Mag.* **72**, 364 (1837).
† B. Powell, *Phil. Trans.* **138**, 213 (1848).

of the beam from the collimator. The continuous spectrum produced by the prism is crossed by bands if the liquid has a smaller refractive index than the glass plate and if the plate is in the thinner part of the prism, whereas if the index of the liquid is greater than that of the plate the latter must be moved to the thicker part of the prism.

The complete analytical investigation of Talbot's bands was accomplished by Airy* and later by Stokes†. Their explanations of the asymmetry involve long and complicated mathematical processes and in consequence the physical aspects of the problem are somewhat obscured.

A simple explanation of the asymmetry based on the ether pulse theory of light was given by Schuster‡. The success of this theory in its application to Talbot's bands gave it strong support when it was a subject of much speculation and controversy.

The suggestion that Talbot's bands might be explained by regarding the film and aperture used for their production as an echelon of two elements was first made by Wood§. Porter‖, following this method of treatment, has given an incomplete and very rough theory of the bands in which the case of almost normal incidence only is considered, and it is concluded that the blackness of the dark bands should increase with diminishing thickness of plate, a result which, as will appear later, a more exact theory shows to be erroneous. For teaching purposes and for experimental application of the bands the author has found these and other accounts existing in the literature of the subject to be inadequate. The theory to be developed, in addition to accounting for Talbot's and Powell's bands and their asymmetrical character, deals with the effect of variation of aperture-width, thickness of plate and angle of incidence of the beam on the echelon. The fact reported by Sethi¶ that in the case of Powell's bands the asymmetry can be reversed by suitable choice of liquid is accounted for, and the criterion for reversal is shown to be equivalent to that stated by Sethi.

§ 2. TALBOT'S BANDS

Normal incidence. Consider the diffraction pattern due to normally incident light of wave-length λ passing through a narrow aperture of uniform width $2s$. The intensity of the beam diffracted in a direction inclined at an angle α to the normal is

$$I_\alpha = A^2 \frac{\sin^2 X}{X^2} \qquad \ldots\ldots(1).$$

where A is a constant depending on the intensity of the incident beam, and X is half the phase-difference between extreme rays of the diffracted beam and $= \lambda^{-1}\pi 2s \sin\alpha$. There is a central maximum given by $X = 0$, i.e. $\alpha = 0$, and other

* G. B. Airy, *Phil. Trans.* **130**, 225 (1840) and **131**, 1 (1841).
† G. G. Stokes, *Phil. Trans.* **138**, 227 (1848).
‡ A. Schuster, *Phil. Mag.* **7**, 1 (1904).
§ R. W. Wood, *Phil. Mag.* **18**, 758 (1909), and *Physical Optics*, p. 254 (1921).
‖ T. Preston, *Theory of Light*, p. 320. (Edited by A. W. Porter, 1928.)
¶ N. K. Sethi, *Phys. Rev.* **16**, 519 (1920).

fainter maxima are given by $X = \pm \frac{1}{2}(2m + 1)\pi$; $m = 0{\cdot}93$, $1{\cdot}96$, $2{\cdot}97$, $3{\cdot}98$, etc. The minima of zero intensity on each side of the central maximum are given by $X = \pm m\pi$, $m = 1, 2, 3$, etc., i.e. $\alpha = \pm \sin^{-1}(m\lambda/2s)$. If one-half of the aperture be covered by a transparent plate of uniform thickness the intensity-distribution is considerably modified, being given as in the case of a two-slit grating by

m

$$I_\alpha = \left(\frac{A \sin X}{X}\right)^2 \left(\frac{\sin 2Y}{\sin Y}\right)^2 \qquad \text{.....(2).}$$

I_a

The first factor is the effect due to a single aperture, with X now equal to $\lambda^{-1}\pi s \sin\alpha$. The second factor represents the effect of interference between corresponding rays from the two elements. Y is half the phase-difference between such rays. If t is the thickness of the plate and μ its refractive index relative to air for

Y

t, μ

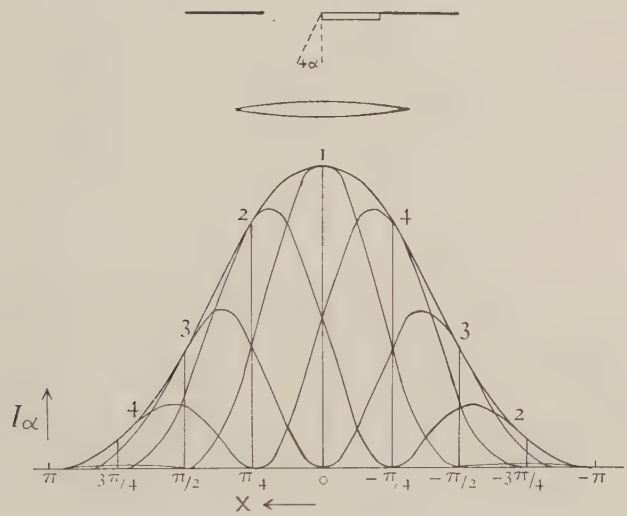

Figure 1. Progression of maxima of a given order with increase of wave-length.

light of wave-length λ, $Y = \pi\lambda^{-1}(\mu t + s\sin\alpha - t\cos\alpha)$. The angle of diffraction α is taken to be positive measured from the normal on the side opposite to the plate. If α is small we may write approximately

$$X = \pi s\alpha/\lambda, \quad Y = \lambda^{-1}\pi(\mu - 1)t + X \qquad \text{.....(3).}$$

The second factor has maxima when $Y = m\pi$, $m = 0, 1, 2, 3$, etc., and is zero when $Y = \frac{1}{2}k\pi$, k being an odd integer.

Equation (2) may be re-written

$$I_\alpha = 4A^2 . \frac{\sin^2 X}{X^2} . \cos^2 Y \qquad \text{.....(4).}$$

The maximum value of the $\cos^2 Y$ is unity, so that with fourfold increase of intensity the single-aperture curve envelopes the interference maxima. If α is small then from the approximate form of (3) the angular separation of successive maxima of $\cos^2 Y$ = the angular separation of successive minima of $\cos^2 Y = \lambda/s$; the separation between a maximum and either of the next minima is $\lambda/2s$. Figure 1 shows several resultant I_α curves for light of slightly different wave-lengths

enveloped by a $X^{-2}\sin^2 X$ curve. In general for a given λ there are two maxima of I_α, but when maxima of I_α and $X^{-2}\sin^2 X$ coincide there is only one, the consecutive I_α maxima coinciding with zero minima of $X^{-2}\sin^2 X$. Thus the spectra may be of double- or single-order type. The separation of double-order I_λ maxima is less than that of maxima of $\cos^2 Y$ and is variable, while the separation of double-order minima is constant and equal to the separation of the single-aperture maximum and either of its adjacent minima. Since the spread of any subsidiary-maximum curve of $X^{-2}\sin^2 X$ is half that of the central-maximum curve, there can be only one maximum of I_λ within it for light of a given wave-length. If the wave-length is such that the central maximum of $X^{-2}\sin^2 X$ and an I_λ maximum coincide, all light of this wave-length is concentrated in that order, for the corresponding maxima of I_α coincide with zero minima between subsidiary maxima of $X^{-2}\sin^2 X$.

For light of a given wave-length, decrease in the thickness of the plate will cause the maximum of any particular order to move in the plate-to-aperture direction, i.e. from right to left in figure 1. The direction of progression of the maxima is the same if the plate-thickness is constant and the wave-length of the incident light is continuously increased. In this latter case we see that $d\alpha\ d\lambda$ is positive for a fixed value of the order m; or $dm\ d\lambda$ is negative in a direction α, figure 2*a*.

Suppose that $Y = m\pi$ for light of wave-length λ_1 in the direction $\alpha = 0$; then if light of this wave-length is incident on the echelon the spectrum will be a single-order one and the corresponding retardation $(\mu - 1)\,t = m\lambda_1$. If light of slightly shorter and longer wave-lengths $\lambda - d\lambda_a$, $\lambda - d\lambda_a$ also is incident, the component intensity distributions will be as indicated in figure 2*a*, the mth-order spectrum of the shorter wave-length being on the plate side of the single-order maximum for λ_1 while the mth-order spectrum of the longer wave-length is on the aperture side. We neglect here the slight change in the spread of the single-aperture curve corresponding to $d\lambda$ and also the effect of the accompanying single-aperture maxima, which relatively are very faint.

Now suppose the aperture and plate to be placed between the collimator and prism of a spectroscope, so that light from the centre of the slit forms a normally incident parallel beam. Let the aperture be on the base side of the prism. Under these conditions the dispersions of echelon and prism, $d\alpha\ d\lambda$ and $d\phi\ d\lambda$, are opposed. Suppose that they are numerically equal: then the component intensity curves are as shown in figure 2*b*. Relative to the single aperture curve for λ_1 those for $\lambda_1 - d\lambda_a$ and $\lambda_1 + d\lambda_a$ are displaced, the former towards the base of the prism, the latter away from it. The mth-order spectra of $\lambda_1 - d\lambda_a$ and $\lambda_1 - d\lambda_a$ approach to coincide with each other and with the mth order of λ_1. The $(m - 1)$th order of $\lambda_1 - d\lambda_a$ and the $(m - 1)$th of $\lambda_1 + d\lambda_a$ are symmetrical in intensity and position with respect to the mth order maxima, the former appearing on the base side, the latter on the edge side, while each is separated from the mth maxima by slightly less than half the distance between single-aperture minima.

The effect of increasing $\lambda_1 + d\lambda_a$ and decreasing $\lambda_1 - d\lambda_a$ by equal amounts is

illustrated in figure 2 *c*, *d*, *e*. In all cases *m*th, $(m + 1)$th and $(m - 1)$th orders coincide and the intermediate zero minima remain unchanged in position. In figure 2*e* the *m*th orders of $\lambda_1 + d\lambda_d$ and $\lambda_1 - d\lambda_d$ are of zero intensity while the $(m + 1)$th of $\lambda_1 - d\lambda_d$ and the $(m - 1)$th of $\lambda_1 + d\lambda_d$ have the greatest intensities

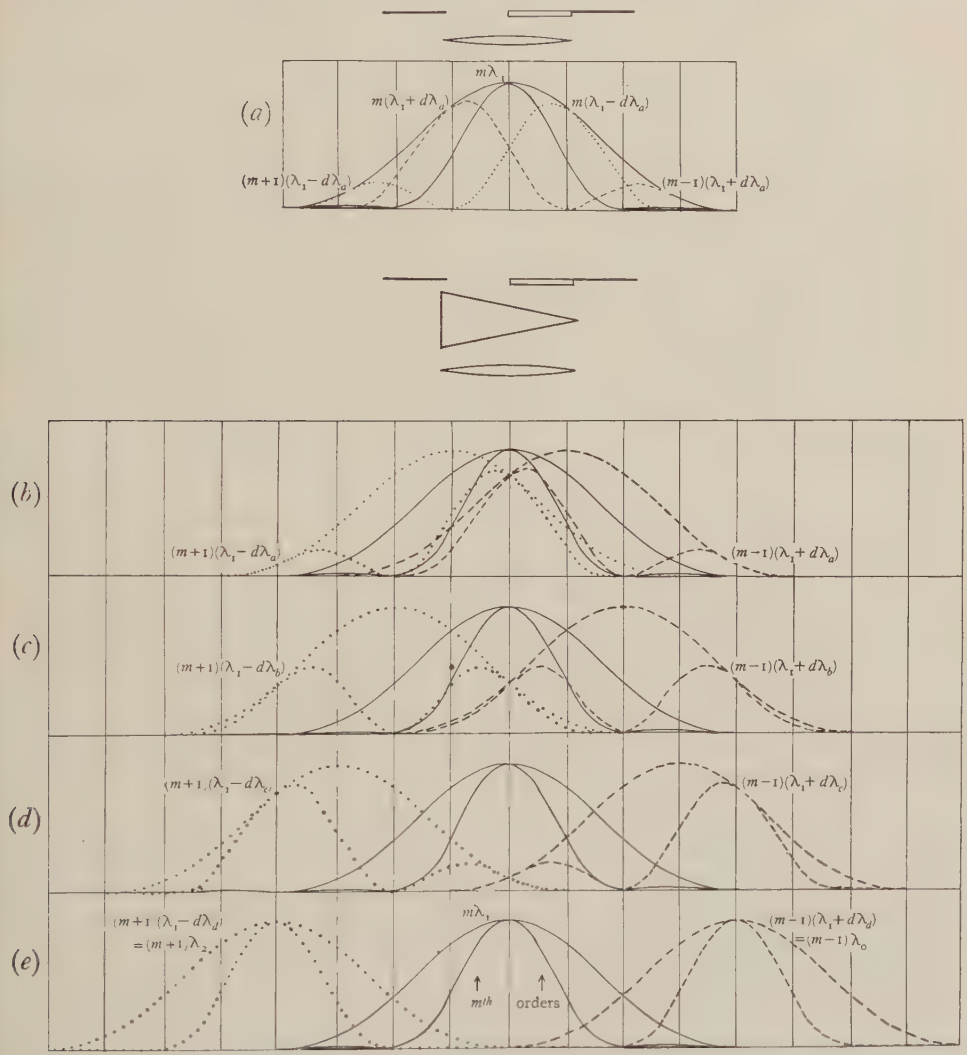

Figure 2. (*a*), intensity-distribution of λ_1, $\lambda_1 + d\lambda_a$, $\lambda_1 - d\lambda_a$ without prism; (*b*, *c*, *d*, *e*), intensity-distributions of light between λ_0 and λ_2 with prism. $\lambda_0 > \lambda_1 > \lambda_2$, $d\lambda_a < d\lambda_b < d\lambda_c < d\lambda_d$.

possible, coinciding with single-aperture maxima. Writing $d\lambda_d = \lambda_1 - \lambda_2 = \lambda_0 - \lambda_1$ and neglecting $d\mu/d\lambda$ we have thus consecutive Talbot bright bands in the spectrum of light between λ_0 and λ_2 at wave-lengths λ_0, λ_1, λ_2 such that $\lambda_0, \lambda_1, \lambda$

$$(m - 1)\lambda_0 = m\lambda_1 = (m + 1)\lambda_2.$$

The separation of these maxima is the same as that between a single-aperture

maximum and the associated first minimum in this region. Talbot dark bands occur at wave-lengths $\lambda_1 + d\lambda_b$ and $\lambda_1 - d\lambda_b$, figure 2*c*, and for them

$$(m - \tfrac{1}{2})(\lambda_1 + d\lambda_b) = (m + \tfrac{1}{2})(\lambda_1 - d\lambda_b).$$

$_1, {}_1\lambda_2, {}_2\lambda_3$ If ${}_0\lambda_1, {}_1\lambda_2, {}_2\lambda_3$ denote wave-lengths for which consecutive dark bands occur,

$$(m - \tfrac{1}{2})_0\lambda_1 = (m + \tfrac{1}{2})_1\lambda_2 = (m + \tfrac{3}{2})_2\lambda_3.$$

Now consider the case in which the plate is on the prism-base side of the aperture. Without the prism the mth order of $(\lambda_1 - d\lambda_a)$ is to the right of the mth order of λ_1 and the mth of $(\lambda_1 - d\lambda_a)$ is to the left, figure 3*a*. Let these spectra be dispersed by means of the prism, still placed with its base to the left. Figures 3*b*, *c*, *d* show an increasing separation of the mth-order spectra of $\lambda_1 - d\lambda$ and $\lambda_1 - d\lambda$ as $d\lambda$ is gradually increased. At the same time the $(m - 1)$th and $(m - 1)$th spectra move in opposite directions, cross each other and attain their maximum intensities in the same positions as in figure 2*e*. It is to be observed that whereas figures 3*c*, *e* are identical with figures 2*c*, *e*, as regards intensity-distribution, figures 3*b*, *d* and 2*b*, *d* are quite different, and because of this difference we should obtain, if figures *b*, *c*, *d*, *e* were superposed in the two cases, intense bright bands separated by zero minima in one case and an almost uniformly intense continuous spectrum in the other. If the illumination is now perfectly continuous the effects are enhanced and we get in one case Talbot's bands and in the other a continuous spectrum in which there is no trace of bands whatever.

It is to be observed that all light of any one of the wave-lengths λ_0, λ_1, λ_2, etc. satisfying the equations $(m - 1)\lambda_0 = (\mu_0 - 1)t$, $m\lambda_1 = (\mu_1 - 1)t$, etc. is concentrated in the corresponding single order. Other orders, for example $(m - 2)$, $(m - 1)$, $(m + 1)$, and $(m + 2)$ of λ_1, coincide with single-aperture zero minima. Spectra outside the range of the single-aperture central maximum are of light intermediate in wave-length between λ_0, λ_1, λ_2, etc. Within the range of any single-aperture subsidiary maximum, there can be only one double-aperture maximum of light of any intermediate wave-length, for the separation of double-aperture minima is the same as that of minima between subsidiary single-aperture maxima. Hence the above argument concerning juxtaposition of light of the same orders when echelon and prism dispersions are equal remains valid when maxima outside the range of the central single-aperture maximum are considered.

The explanation of Talbot's bands given by Walker* and to which reference has been made recently by Milne† must be regarded as incomplete and unsatisfactory since no account is taken of the production of double-order spectra by the plate and aperture.

Incidence not normal. The effect of rotating the echelon about a vertical axis when it is so placed as to produce bands is to alter the values of X and Y, and a shift of the bands across the spectrum results. The appearance or non-appearance of the bands is not dependent on the angle of incidence of the beam.

* J. Walker, *Phil. Mag.* **11**, 531 (1906).
† J. R. Milne, Discussion on paper by A. C. G. Beach, *Proc. Phys. Soc.* **45**, 474 (1933).

If the echelon is placed between the prism and telescope the above method of explanation still applies. In the former case the angle of incidence on the echelon is the same for all wave-lengths but the angle of incidence on the prism depends on the wave-length, while in the latter case the angle of incidence on the prism is the same for all wave-lengths but that on the echelon varies with wave-length.

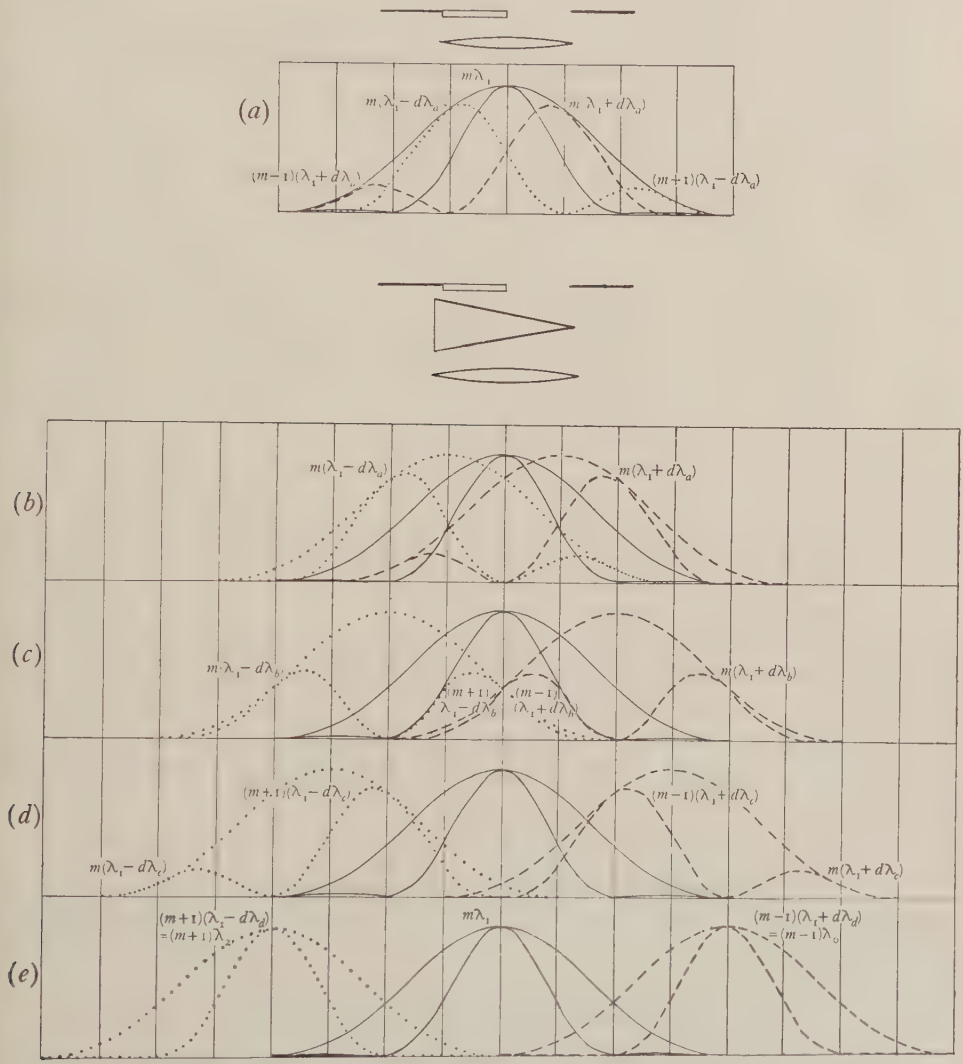

Figure 3. Intensity-distributions corresponding to figure 2 but with echelon reversed.

The case of the echelon between prism and telescope is the one dealt with by Porter*, to whose treatment reference has already been made. We proceed to the analytical deduction of the condition to be satisfied if the bands are to appear.

* *Loc. cit.*

The conditions obtaining involve the theory of the tilted echelon, which has been treated very fully by Galitzin* and presented in a simplified form by Williams†.

Adopting Williams's notation,

ψ let ψ be the angle of incidence on the echelon of light of wave-length λ, positive in figure 4;

θ θ the angle of refraction in the plate;

α α the angle of deviation of the diffracted beam, positive in figure 4;

μ μ the refractive index of plate relative to air for light of wave-length λ;

t t the thickness of the plate; and

s s the width of the aperture and of the plate.

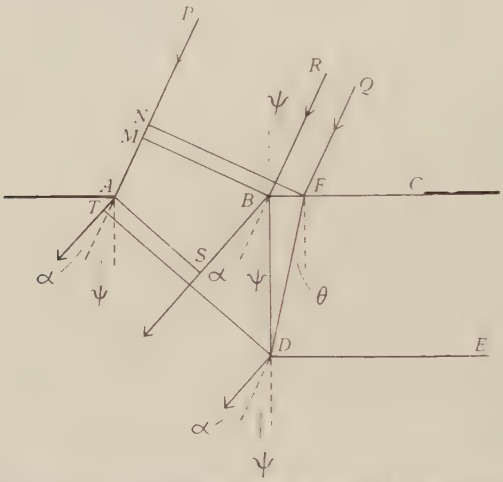

Figure 4. Diffraction of parallel beam by echelon: MB, AS and NF, TD, corresponding wave-fronts before and after transmission.

The intensity I_α of light of which the deviation is α is given by equations (2) and (4). In the present case X is half the phase-difference corresponding to $BS - MA$ and

$$X = \lambda^{-1}\pi s \{\sin(\psi + \alpha) - \sin\psi\} \quad \ldots\ldots(5),$$

Y is half the phase-difference corresponding to $\mu FD - NA - AT$ and

$$Y = \lambda^{-1}\pi t \{\mu \sec\theta - \tan\theta \sin\psi - \cos(\psi + \alpha)\} + X \quad \ldots\ldots(6).$$

We proceed to show that, as in the case of normal incidence ($\psi = 0$) $d\alpha/d\lambda$ is positive whether $d\psi/d\lambda$ be positive or negative.

For the mth order of interference of light of wave-length λ,

$$m\lambda = t\{\mu \sec\theta - \tan\theta \sin\psi - \cos(\psi + \alpha)\} + s\{\sin(\psi + \alpha) - \sin\psi\} \quad \ldots\ldots(7),$$

or, since $\sin\psi = \mu \sin\theta$

$$m\lambda = t\{\sqrt{(\mu^2 - \sin^2\psi)} - \cos(\psi + \alpha)\} + s\{\sin(\psi + \alpha) - \sin\psi\} \quad \ldots\ldots(8),$$

the root sign being essentially positive.

* F. B. Galitzin, *Bull. Acad. Sc. de St Pétersbourg*, **31**, 67 (1905).

† W. E. Williams, *Proc. Opt. Convention*, Part 2, p. 987 (1926).

After differentiation with respect to λ and arrangement of terms,

$$\frac{m}{t} - \frac{\mu\, d\mu/d\lambda}{\sqrt{(\mu^2 - \sin^2\psi)}} = \frac{d\psi}{d\lambda}\left[\sin(\psi+\alpha) + \frac{s}{t}\cos(\psi+\alpha) - \cos\psi - \frac{\sin\psi\cos\psi}{\sqrt{(\mu^2-\sin^2\psi)}}\right] + \frac{d\alpha}{d\lambda}\left[\sin(\psi+\alpha) + \frac{s}{t}\cos(\psi+\alpha)\right] \quad \ldots\ldots(9).$$

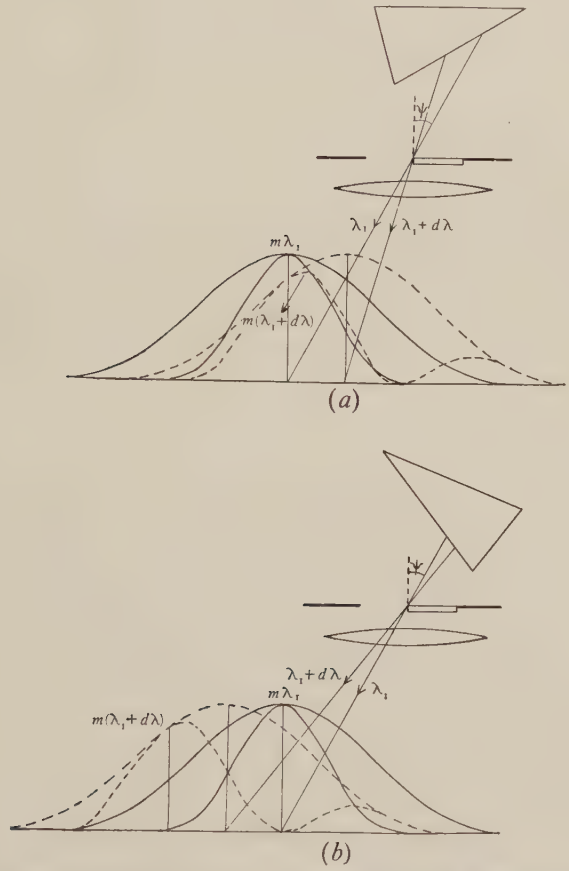

Figure 5. Echelon between prism and telescope. Intensity curves of mth orders of λ_1 and $\lambda_1 + d\lambda$. (*a*), $d\psi/d\lambda$ negative; (*b*), $d\psi/d\lambda$ positive. In both (*a*) and (*b*) $d\alpha/d\lambda$ is positive and numerically equal to $d\psi/d\lambda$.

Since $d\mu/d\lambda$ is negative, the left-hand side of equation (9) is positive. The coefficient of $d\alpha/d\lambda$ exceeds that of $d\psi/d\lambda$ by $(s/t)\cos\psi + \sin\psi\cos\psi/\sqrt{(\mu^2 - \sin^2\psi)}$, so that whether $d\psi/d\lambda$ be negative or positive $d\alpha/d\lambda$ is positive. In the case of normal incidence for all wave-lengths, previously treated, $\psi = 0$, and $d\psi/d\lambda = 0$, $d\alpha/d\lambda$ is positive; compare equation (3).

In figures 5 *a* and *b* are shown the dispositions of prism and echelon for which $d\psi/d\lambda$ is negative and positive respectively, $d\psi/d\lambda$ is now identical with $d\phi/d\lambda$, the angular dispersion of the prism, and if it is numerically equal to the dispersion of the echelon while the mth maximum of λ_1 is supposed to be in the single-order

position, we have intensity-distribution curves comparable with those in figures 2*b* and 3*b*. The explanation of the asymmetry is seen to follow as before by reference to figures 2 and 3.

In terms of the present notation Porter* finds the condition that I shall be unchanged when light of wave-length λ and incident at a small angle ψ is replaced by light of wave-length $\lambda + d\lambda$ incident at an angle $\psi + d\psi$, where the sign convention for ψ is opposite to that in the present notation—the condition, that is, that the mth order of λ shall coincide with the mth order of $(\lambda + d\lambda)$. This he finds to be $d\psi/d\lambda = (\mu - 1)\, t/\lambda s$. The explanation of the asymmetry is based on the fact that $d\psi/d\lambda$ must be positive. He points out that, even if the dispersions of echelon and prism are equal and opposite so far as the range of wave-length $d\lambda$ is concerned, perfect blackness of the dark bands will not in general be obtained. Light of wave-length λ is supposed to be incident normally on the echelon, and $m\lambda = (\mu - 1)\, t$. By using the above value of $d\psi/d\lambda$ it is concluded that in order that the $(m + 1)$th orders of λ and $(\lambda + d\lambda)$ shall coincide m must be equal to 1, and a very thin strip would be required to make adjacent maxima correspond. In practice the author has found that with a glass plate 0·016 cm. thick and orders between 140 and 200 the dark bands may be very dark indeed provided that a suitable value of s is chosen. Doubt†, using a spectrograph of high resolving-power, has observed bands with a glass plate more than 9 cm. thick. Reference to figure 2 shows that provided that (i) the opposed dispersions of echelon and prism are equal for light whose wave-length lies between the wave-lengths λ_0 and λ_1 to which correspond consecutive bright bands, and (ii) the spread of the single-aperture curves for these wave-lengths is the same, the intervening dark bands should be perfectly black. According to Porter, for perfect blackness of dark bands within the range $d\lambda$ it is required that the interval between two successive maxima of I_λ should be constant for the range. From figure 2 it is seen that, while the intervals between successive maxima within the ranges λ_0 to λ_1, and λ_1 to λ_2 are not constant, perfect blackness is obtained since the intervals between successive minima are constant and it is supposed that these ranges are so small that the differences in spread of the corresponding single-aperture curves are negligibly small. Writing $m\lambda_1 = (m + 1)\lambda_0$ we see that $(\lambda_0 - \lambda_1)$ is inversely proportional to m, and so condition (ii) is the more nearly satisfied the smaller is $(\lambda_0 - \lambda_1)$ and, therefore, the greater is the value of m and the greater is the thickness of the plate. Even if the condition for perfect blackness were that the interval between two successive maxima should be constant, it can be shown that perfect blackness would be approached as the thickness increased, a conclusion directly opposite to that arrived at by Porter.

Suppose that light of wave-lengths λ and $(\lambda + d\lambda)$ is incident on the echelon at angles ψ and $(\psi + d\psi)$ respectively. Let the effect of the prism, so placed as to produce Talbot's bands, be to make the mth order maxima of λ and $(\lambda + d\lambda)$ coincide. We proceed to determine the conditions, so far as the plate-thickness t and the angle of incidence ψ are concerned, that the separation of the $(m + 1)$th-order maxima of λ and $(\lambda + d\lambda)$ shall be as small as possible.

* *Loc. cit.*

† T. E. Doubt, *Phys. Rev.* **10**, 322 (1917).

For small angles such that $\sin\beta = \beta$ and $\cos\beta = 1 - \frac{1}{2}\beta^2$ equation (8) assumes the form β

$$m\lambda = t\{\mu - 1 + \tfrac{1}{2}(\psi + \alpha)^2 - \psi^2/2\mu\} + s\alpha \qquad \text{.....(10).}$$

If the mth order of λ is single as in figure 2*a*

$$m\lambda = t\{\mu - 1 + \tfrac{1}{2}\psi^2(1 - 1/\mu)\} \qquad \text{.....(11).}$$

If α_1 is the angular separation of the mth and $(m + 1)$th orders of λ α_1

$$(m + 1)\lambda = t\{\mu - 1 + \tfrac{1}{2}(\psi + \alpha_1)^2 - \psi^2/2\mu\} + s\alpha_1 \qquad \text{.....(12).}$$

From (11) and (12)

$$\alpha_1 = \frac{\lambda}{s + (\psi + \frac{1}{2}\alpha_1)\,t} \qquad \text{.....(13).}$$

Hence when ψt is appreciable compared with s the separation of successive maxima is less than that (λ/s) between a central single-aperture maximum and an adjacent minimum, so that three orders appear in the so-called single-order position.

By differentiation of equation (13) with respect to λ we obtain an expression for the separation $d\alpha_1$ of the $(m + 1)$th orders of λ and $\lambda + d\lambda$

$$\frac{d\alpha_1}{d\lambda} = \frac{1 - \alpha_1 t\, d\psi/d\lambda}{s + \psi t + \alpha_1 t} \qquad \text{.....(14).}$$

When the echelon is between collimator and prism, $d\psi/d\lambda = 0$ and $d\alpha_1$ decreases as t or ψ increases. Light of wave-lengths λ and $(\lambda + d\lambda)$ and of the same order will be deviated by different amounts. If the difference is $d\phi$ then, for coincidence of the mth and of the $(m - 1)$th orders of λ and $(\lambda + d\lambda)$, $d\alpha_1 = -d\phi$. The smaller $d\alpha_1$ the more nearly will this condition be fulfilled for light of intermediate wave-lengths. Hence perfect blackness is approached as t or ψ increases.

When the echelon is between prism and telescope, the angle of incidence on the echelon varies with the wave-length and $d\psi/d\lambda$ is identical with the angular dispersion of the prism. For coincidence of corresponding orders $d\alpha_1/d\lambda = -d\psi/d\lambda$. Then $d\alpha_1/d\lambda = 1/(s + \psi t)$ and again perfect blackness within the range $d\lambda$ is approached as t or ψ increases.

Considering the entire visible spectrum, the visibility of bands will vary since the dispersions of echelon and prism vary considerably in this wide range. The change in visibility is to be attributed to deviation from condition (i) above rather than to deviation from condition (ii).

The optimum value of s, the aperture-width for a given plate of thickness t, is found by equating echelon and prism dispersions. Denote the refractive indices of plate and prism materials by μ_e and μ_p. If T is the effective thickness of the prism for a beam of width $2s$ the prism dispersion $d\phi/d\lambda$ is $(T/2s)\,d\mu_p/d\lambda$. The echelon dispersion $\dfrac{d\alpha}{d\lambda} = \dfrac{m}{s} - \dfrac{t}{s}\dfrac{d\mu_e}{d\lambda}$ when the incidence is normal. Hence for wave-lengths in the neighbourhood of λ the best value of s is determined by μ_e, μ_p, T s

$$t\left(\mu_e - 1 - \lambda\frac{d\mu_e}{d\lambda}\right) + s\alpha = \frac{T}{2}\lambda\frac{d\mu_p}{d\lambda} \qquad \text{.....(15).}$$

If consecutive bright bands occur at wave-lengths λ_0, λ_1, λ_2, etc. we may write *approximately*, as on p. 615,

$$(m - 1)\lambda_0 = m\lambda_1 = (m + 1)\lambda_2 = \text{etc.}$$

The angle of diffraction α of light of these wave-lengths will be zero, and by equation (8)

$$(m - 1)\lambda_0 = t\{\sqrt{\mu_0^2 - \sin^2\psi_0) - \cos\psi_0}\}$$

$$m\lambda_1 = t\{\sqrt{(\mu_1^2 - \sin^2\psi_1) - \cos\psi_1}\} \qquad \text{.....(16),}$$

exactly, where μ_0, μ_1, ψ_0, ψ_1, correspond to λ_0, λ_1. If the echelon is between collimator and prism, ψ is the same for all wave-lengths.

When the echelon is placed between the eye and the eyepiece of the spectroscope the plate must be placed on the side on which the violet appears, with the aperture towards the red. Consideration of the elementary theory of the eyepiece shows that in this case $d\psi/d\lambda$ is negative. If the echelon is reversed left and right $d\psi/d\lambda$ is positive and the asymmetry is readily accounted for as above.

The author has observed that when the echelon is placed in front of the spectroscope slit and illuminated by parallel white light so that the shadow of the edge of the plate falls on the slit, bands having the same appearance and spacing as Talbot's appear. Apart from a reduction in intensity of the bright bands there is no change in the appearance of the bands when the echelon is removed from a position immediately in front of the slit to a distance of one metre, the shadow of the plate-edge remaining on the slit. These bands are not subject to asymmetry of the Talbot type. If, however, the light transmitted by the echelon is focused on the slit, the bands may or may not exhibit asymmetry, according to the position of the image with respect to the edges of the slit. If the slit is so narrow that it does not admit the whole of the light corresponding to the single-aperture central diffraction maximum, the bands appear when the plate is either to the left or to the right of the slit. The same is true if the slit is wide and the chief diffraction maximum is partly intercepted by one side of the slit, but if the whole of this light is admitted the bands appear only when the plate is on the prism-base side of the slit.

To explain these effects it is necessary to refer to the fact that light of wave-lengths λ_0, λ_1, λ_2, ... corresponding to the centres of bright Talbot bands is in each case concentrated entirely in one single-order maximum. Such light is transmitted by echelon and collimator slit without deviation. Light of other wave-lengths is deviated right and left in the directions of the single-aperture subsidiary maxima, and is intercepted if the slit is not very wide.

In the case in which the Fraunhofer diffraction pattern is focused on the slit and the slit is wide enough to admit the main part of it the conditions are similar to those to which figures 2 and 3 refer, with the difference that the collimator lens produces a left-and-right inversion of the diffracted beams incident on the prism and the echelon must be placed with the plate on the prism-base side if the bands are to appear.

§ 3. OBLIQUE BANDS PRODUCED WHEN ECHELON AND PRISM DISPERSIONS ARE AT RIGHT ANGLES

If the echelon is placed between collimator and prism with the plate-edge horizontal and the slit is replaced by a small point aperture, the diffraction pattern observed consists of short bright and dark bands inclined to the horizontal at an angle which depends on the relative dispersions of echelon and prism. If the plate is below the aperture the bands are inclined upwards in the violet-to-red direction; if it is above, the inclination is downwards in the same direction. Corresponding to each bright band there are several fainter and shorter maxima similarly inclined. Oblique bands of this type have been photographed by Sethi* and by Beach*. The latter has accounted for the bands by means of vector diagrams. A very direct

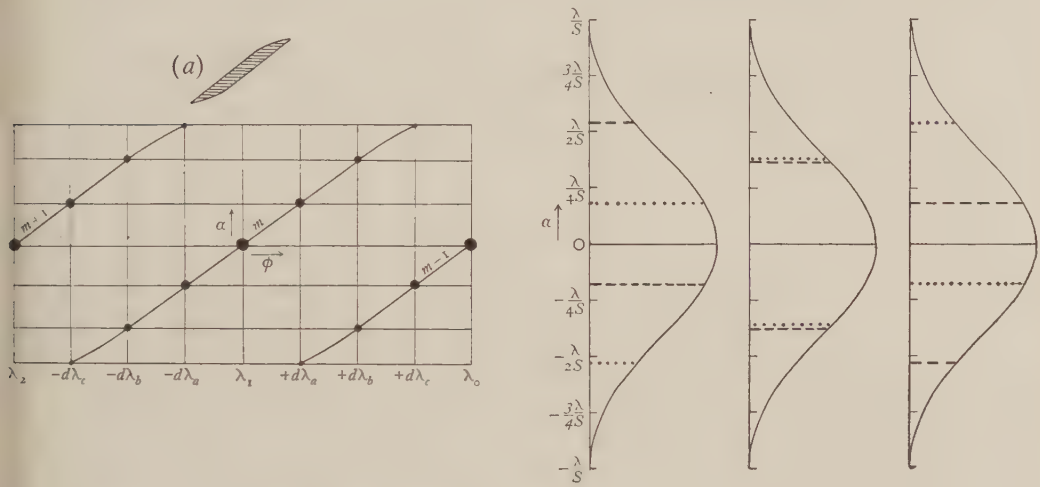

Figure 6. Echelon and prism dispersions at right-angles. (*a*) shape of main bright bands.

graphical representation of the mode of production of these bands is afforded by the method described in § 2 of this paper. In figure 6 the relative positions and intensities of maxima of I_α for various wave-lengths are shown, the echelon dispersion being vertical. On the left of the figure the resultant positions of the maxima after horizontal dispersion by the prism are represented by points whose sizes indicate the relative intensities. The actual shape of the main bands is shown at (*a*). The curvature near the ends is seen at once to be due to the fact that the rate of increase of vertical displacement of maxima of I_α diminishes appreciably near single-aperture minima.

§ 4. POWELL'S BANDS

The explanation of Powell's bands apparently is identical with that of Talbot's bands. Sethi*, however, using Stokes's* modified method of producing Powell's bands, obtained results which, he considered, showed that "the side of the cell on

* *Loc. cit.*

which the glass plate should be immersed relative to the prism is determined not by the relation between the refractive indices of liquid and plate but by the group velocities in the two media. Owing to the highly dispersive nature of the liquid employed in the experiments (a mixture of carbon disulphide and benzene), the position of the plate has on this account to be changed from one side to the other at a stage at which the refractive index of the liquid is still lower than that of the plate for the whole of the visible spectrum."

In further consideration of this reversal of Talbot asymmetry, let us examine the rate of change with wave-length of the order of interference between two corresponding normally incident rays of the same wave-length in air, λ_a, in the direction $\alpha = 0$, first when the media are glass and air, as for Talbot's bands, and then when the media are glass and liquid, as for Powell's bands. We shall assume the air to be non-dispersive and of unit refractive index.

λ_a

Talbot's bands. $\frac{d}{d\lambda_a}$ (order of retardation of λ by glass—order of retardation of λ by air)

$$= \frac{d}{d\lambda}\frac{t}{\lambda}(\mu_g - 1)$$

$$= \frac{t}{\lambda_a^2}\left\{\lambda_a \frac{d\mu_g}{d\lambda_a} - (\mu_g - 1)\right\} \qquad \text{.....(17).}$$

Thus, since $d\mu_g/d\lambda_a$ is negative, $dm/d\lambda_a$ is negative; that is, the greater the wave-length the smaller is the order-difference. Figures 2*a* and 3*a* are seen to be consistent with this result, for the order of $(\lambda_1 - d\lambda_a)$ that coincides with $m\lambda_1$ is less than m while the corresponding order of $(\lambda_1 - d\lambda_a)$ is greater than m.

Powell's bands. $\frac{d}{d\lambda_a}$ (order of retardation of λ_a by glass—order of retardation of λ_a by liquid)

$$= \frac{d}{d\lambda_a}\left\{\frac{t}{\lambda_a}(\mu_g - \mu_l)\right\}$$

$$= \frac{t}{\lambda_a^2}\left\{\lambda_a\left(\frac{d\mu_g}{d\lambda_a} - \frac{d\mu_l}{d\lambda_a}\right) - (\mu_g - \mu_l)\right\} \qquad \text{.....(18).}$$

This may be negative or positive. For a negative value

$$\mu_g - \mu_l > \lambda_a\left(\frac{d\mu_g}{d\lambda_a} - \frac{d\mu_l}{d\lambda_a}\right) \qquad \text{.....(19),}$$

$d\mu_g/d\lambda_a$ and $d\mu_l/d\lambda_a$ are negative so that if the numerical value of $d\mu_l/d\lambda_a$ > the numerical value of $d\mu_g/d\lambda_a$ and $\mu_g > \mu_l$, then $dm/d\lambda_a$ is negative and the asymmetry is of the Talbot type.

But if $d\mu_l/d\lambda_a > d\mu_g/d\lambda_a$ numerically, $dm/d\lambda$ may be positive for a certain value of $\mu_g - \mu_l$ such that

$$\mu_g - \mu_l < \lambda_a\left(\frac{d\mu_g}{d\lambda_a} - \frac{d\mu_l}{d\lambda_a}\right) \qquad \text{.....(20).}$$

In this case the progression of maxima as the wave-length increases is opposite to that indicated in figure 1 and the asymmetry is reversed.

The condition expressed by (20) for reversal of asymmetry is equivalent to that implied by Sethi*, which is that the group velocity of light in glass shall exceed the group velocity in liquid.

The group velocity, U, $= V_m - \lambda_m \, dV_m/d\lambda_m$, where V_m is the wave velocity of light of wave-length λ_m in medium m. U, V_m λ_m, m

If V_0 is the velocity of light *in vacuo*,

$$V_m = \frac{V_0}{\mu_m}.$$

$$U = \frac{V_0}{\mu_m} + \lambda_m \frac{V_0}{{\mu_m}^2} \frac{d\mu_m}{d\lambda_m}$$

$$= V_0 \left(\frac{\mu_m + \lambda_m \, d\mu_m/d\lambda_m}{{\mu_m}^2} \right).$$

Hence the condition that the group velocity in glass shall exceed that in liquid is

$$\frac{\mu_g + \lambda_g \, d\mu_g/d\lambda_g}{{\mu_g}^2} > \frac{\mu_l + \lambda_l \, d\mu_l/d\lambda_l}{{\mu_l}^2} \quad \text{.....(21).}$$

Since $\mu_m = \dfrac{\lambda_a}{\lambda_m}$ and $\dfrac{d\mu_m}{d\lambda_m} = \dfrac{\lambda_a}{\lambda_m (d\lambda_a/d\mu_m - \lambda_m)}$ this condition on further reduction becomes identical with equation (20).

* *Loc. cit.*

531.717.1 535.321

A DETERMINATION BY SPECTROMETER OF THE METRICAL THICKNESS AND DISPERSIVE POWER OF A THIN FILM

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ABSTRACT. By counting the numbers of Edser-Butler and Talbot bands produced within the same spectral range by a thin film of glass and a prism spectrometer, the metrical thickness of the film can be determined. The refractive index of the film for light of any standard wave-length can then be calculated from a count of the number of Talbot bands passing the position of the corresponding line in the spectrum as the Talbot echelon is rotated about the vertical edge of the film through an accurately measured angle. Two spectrometers are used in conjunction as a double-table spectrometer, and the other apparatus required is of the usual student-laboratory type. In the case of a film about 0·016 cm. thick the thickness is measured with an error not exceeding 0·00004 cm. and refractive indices are measured with an error not exceeding 0·0013.

§ 1. INTRODUCTION

μ, t

In the usual interferometer determinations of film-thickness the optical thickness, that is the product of refractive index μ and metrical thickness t, is measured. One method of determining t independently of μ in the case of a thin film whose thickness is of the order 0·01 cm. is to form an air wedge by placing the film between two accurately worked optical flats and to determine the angle of the wedge by measuring the separation of fringes formed by the interference of monochromatic light reflected at the wedge-surfaces. If it is required that the error of measurement be not more than 0·5 per cent, precision apparatus similar to that described by Smith* is necessary.

In the method to be described the thickness is determined by counting the numbers of Edser-Butler and Talbot fringes formed by the film between two well-separated spectral lines. The variation of refractive index with wave-length is measured by counting the number of fringes passing a spectral line as the Talbot film and aperture are rotated about a vertical axis through an angle which can be measured accurately. All the apparatus required is of student-laboratory type.

The method is based on the theory of the tilted transmission echelon†, for the film and aperture may be regarded as an echelon of two spaces‡.

* C. F. Smith, *Machinery*, October 27 (1927).

† F. B. Galitzin, *Bull. Acad. Sc. de St Petersbourg*, 31, 67 (1905); W. E. Williams, *Proc. Opt. Convention*, 987 (1926).

‡ See p. 612.

§ 2. THEORY

Suppose the echelon to be placed between spectrometer collimator and prism in such a manner as to produce Talbot's bands, and let it be capable of rotation about a vertical axis through a measurable angle so that the angle of incidence of the beam of white light from the collimator upon the echelon may be varied and measured.

In figure 1 AB and $BCED$ represent the aperture and film respectively. Let PA and QF be corresponding rays of the incident beam; then if the relative retardation of these rays in the undeviated direction, after transmission, is an integral multiple of some wave-length λ_0 there will be a bright Talbot band corresponding to this wave-length in the spectrum*. If FM is the incident wave-front and DN the parallel diffracted wave-front the retardation is

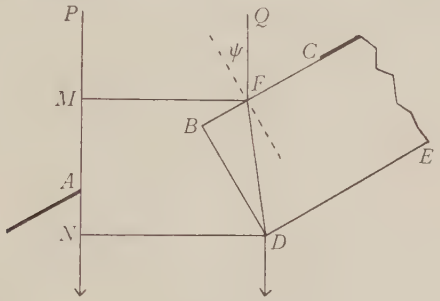

Figure 1.

$$(\mu_0 FD - MN), \text{ which } = t\sqrt{(\mu_0^2 - \sin^2\psi)} - t\cos\psi$$

where t is the thickness of the film; t

μ_0 the refractive index of film relative to air for light of wave-length λ_0; and μ_0, λ_0

ψ the angle of incidence. ψ

Hence we may write $m\lambda_0 = t\sqrt{(\mu_0^2 - \sin^2\psi)} - t\cos\psi$.

If the next bright band occurs at λ_1 λ_1

$$(m+1)\lambda_1 = t\sqrt{(\mu_1^2 - \sin^2\psi)} - t\cos\psi$$

and for the nth bright band

$$(m+n)\lambda_n = t\sqrt{(\mu_n^2 - \sin^2\psi)} - t\cos\psi \qquad \text{.....(1),} \qquad \lambda_n$$

μ_n being the refractive index corresponding to λ_n. μ_n

Between λ_0 and λ_n there will be n band-widths and

$$n = t\left\{\frac{\sqrt{(\mu_n^2 - \sin^2\psi)} - \cos\psi}{\lambda_n} - \frac{\sqrt{(\mu_0^2 - \sin^2\psi)} - \cos\psi}{\lambda_0}\right\} \qquad \text{.....(2).}$$

If $$\psi = 0, \quad n = t\left\{\frac{\mu_n - 1}{\lambda_n} - \frac{\mu_0 - 1}{\lambda_0}\right\} \qquad \text{.....(3).}$$

* O. Darbyshire, the preceding paper (p. 611).

If Edser-Butler fringes are formed by placing the film in front of the spectrometer slit normal to the collimator axis and illuminating it by normally incident white light, the number p of fringe-widths between λ_0 and λ_n is given by

p

$$p = 2t\,(\mu_n/\lambda_n - \mu_0/\lambda_0) \qquad \text{.....(4).}$$

Eliminating μ_n and μ_0 from equations (3) and (4) we have

$$t = \frac{p - 2n}{2\,(\lambda_n^{-1} - \lambda_0^{-1})} \qquad \text{.....(5).}$$

For normal incidence and a bright Talbot band at λ_s

m_s, μ_s

$$m_s\lambda_s = t\,(\mu_s - 1) \qquad \text{.....(6).}$$

Suppose the echelon to be rotated about the vertical edge of the film till the angle of incidence is ψ, and let the number of Talbot bands crossing the position λ_s in the spectrum be q; then

λ_s

q

$$\begin{aligned}(m_s + q)\,\lambda_s &= t\,\{\sqrt{(\mu_s^2 - \sin^2\psi)} - \cos\psi\} \\ &= t\,[\sqrt{\{(1 + m_s\lambda_s/t)^2 - \sin^2\psi\}} - \cos\psi].\end{aligned}$$

Squaring and simplifying the last equation we get

$$m_s = \frac{q^2\lambda_s + 2qt\cos\psi}{2t\,(1 - \cos\psi) - 2q\lambda_s} \qquad \text{.....(7).}$$

By means of equations (5), (6) and (7), t, μ_s and m_s can be evaluated.

§ 3. EXPERIMENTAL DETAILS

The film used was a piece of microscope cover-glass less than 0·02 cm. thick. Preliminary tests showed that over the area to be used the thickness was uniform and that Edser-Butler fringes of good visibility could be produced by transmission from a pointolite lamp of a parallel beam of white light incident normally on the film when the latter was placed immediately in front of the spectrometer slit. Talbot bands of a high degree of visibility were obtained when the widths of aperture and film-space were each about 1·3 mm. and the length 5 mm. The Edser-Butler fringes were so closely spaced that an accurate count by direct observation was tedious and difficult. The spacing of the Talbot bands was about six times that of the Edser-Butler fringes, but again a direct count of the number between standard spectral lines could not be effected as the standard lines were split into displaced component spectra by the echelon. Consequently both Edser-Butler and Talbot bands were photographed by means of a small constant-deviation spectrograph, and after exposure in each case the echelon was removed and a mercury-arc comparison spectrum photographed. To facilitate counting the numbers of bands between standard lines on the photographic plate a low-power travelling microscope was used. It was found possible to estimate to one-tenth of a band-width.

For the determination of orders of Talbot bands and refractive indices two good spectrometers of the usual student type were used. The usual adjustments were first made and then the two instruments were so placed as to act as a double-table

spectrometer, the unrequired telescope of one and the unrequired collimator of the other being turned aside, figure 2.

The echelon was mounted on a simple metal holder *MH* to be placed on the table T_1 of the spectrometer whose collimator was to be used. The film-edge *E* was adjusted for coincidence with the axis of rotation of the table by viewing it through a fixed low-power microscope and moving the holder until the edge appeared perfectly stationary when the table was rotated. On the table T_2 of the other spectrometer was a prism in the position of minimum deviation for the mercury green line. The telescope of this spectrometer was used for the observation of the bands. A pointolite lamp *L* was the source of white light, and the slit *S* could be illuminated by the mercury arc *A* by reflection from a glass plate *P*.

By use of the mercury lines and several other lines produced by salts in a Bunsen flame, a graph of telescope vernier readings against wave-length was first obtained. This was necessary because a bright band was not generally in exact coincidence

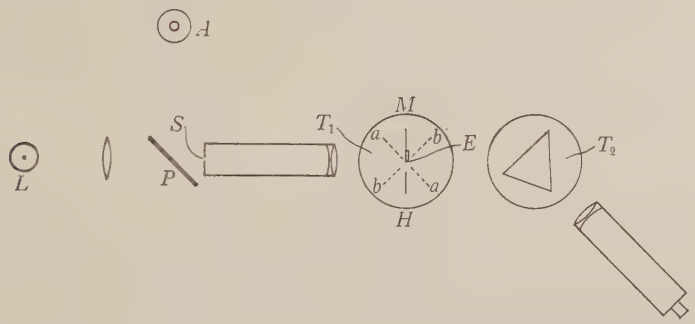

Figure 2.

with a standard line. From the difference of vernier readings for a standard line and the nearest bright band the corrected wave-length at which the bright-band-centre occurred could be estimated.

Rotation of the table and echelon clockwise from the position *aa*, figure 2, produced a motion of the bands across the spectrum from red to violet until the echelon was in the position of normal incidence when the motion stopped. Further rotation in the same direction towards the position *bb* caused the bands to cross the spectrum from violet to red. The position of the table corresponding to the stationary position of the bands was rather indefinite and in the measurement of the angle of incidence ψ the corresponding vernier reading was not used. To determine the order m_s of a bright band for normal incidence and occurring at some position λ_s in the spectrum by means of equation (7) it was necessary to measure the angle of incidence when from a count of the number q of bands passing the cross-wire the order of a band at the same position was known to be $(m_s + q)$. The cross-wire was set on the band in the stationary position. The table was turned from its position for normal incidence and stationary bands, first clockwise to *bb*, then counter-clockwise to *aa*, and the vernier was read in each case when the qth band passing in each case coincided with the cross-wire. Half the difference between the two readings was the angle ψ corresponding to q.

λ_s

m_s

§ 4. RESULTS

Determination of t

p, number of Edser-Butler band-widths between λ_0 (5461 A.) and λ_n (4358 A.) = 231·0.

n, number of Talbot band-widths between λ_0 (5461 A.) and λ_n (4358 A.) = 42·5.

$$t = \frac{p - 2n}{(\lambda_n^{-1} - \lambda_0^{-1})} = 0{\cdot}01576 \text{ cm.}$$

Table 1. Determination of m_s and μ_s

Standard line λ (Å.)	Corrected λ_s (Å.)	Number of bands q	Angle of incidence ψ (Deg. min. sec.)	Calculated order of band	Correct integral order m_s	Refractive index of film for light of wave-length λ_s μ_s
5791	5772	20	36 0 0	143·7	144	1·5274
		30	43 23 0	143·6		
		35	46 28 30	143·8		
		40	49 18 30	144·0		
5461	5453	20	34 58 45	153·3	153	1·5293
		30	42 10 30	153·4		
		35	45 14 15	153·2		
		40	48 1 0	153·2		
4916	4921	20	33 16 0	171·2	171	1·5339
		30	40 10 15	171·1		
		35	43 7 45	170·6		
		40	45 48 15	170·8		
4608	4603	20	32 11 45	183·6	184	1·5374
		30	38 53 0	184·1		
		35	41 45 30	183·7		
		40	44 21 45	183·8		
4358	4350	20	31 16 30	195·9	196	1·5410
		30	37 49 0	196·1		
		35	40 35 45	196·2		
		40	43 9 0	196·2		

n and p were measurable to one-tenth of a band-width so that t was accurately determined to 0·25 per cent. For a particular value of λ_s, the estimated values of m_s for four different values of q were never more than 0·4 greater or less than the same integer, so that the integral value of m_s was determined with certainty. The values of μ_s were thus correct within ± 0·0013 and fitted a smooth μ_s, λ_s curve and a straight μ_s, λ_s^{-2} line very well; from either of these graphs the dispersive power of the film in any region of the spectrum could be determined.

534·323

LOGARITHMIC UNITS: A NEED IN ACOUSTICS

By A. H. DAVIS, D.Sc.

Received March 15, 1934.

ABSTRACT. In view of the confusion which prevails in the use of logarithmic units, the paper suggests the use of a new unit, to be named the "brig", for the ratio of two quantities, together with certain subsidiary changes, particularly in the nomenclature of acoustics.

THERE appears to be an urgent need for a convenient terminology for use when quantities are expressed in common logarithmic form. To meet the difficulty different branches of science tend to adopt special nomenclatures of their own, and although in some cases the requirements of the workers concerned may be met adequately, in almost all the meaning would be clearer to laymen if a common terminology were available. The name of Henry Briggs, the inventor of common logarithms, has a strong claim to adoption for the name of the unit*, and the following definition is therefore put forward: Two numbers N_1 and N_2 are said to differ by n brigs or $10n$ decibrigs when $n = \log_{10}(N_1/N_2)$.

On the above definition a 10-fold change in any quantity would be called a change of 1 brig; a change of 10^n-fold would be known as n brigs or $10n$ decibrigs, and so on. Wherever logarithmic expression is more appropriate or more vivid than other forms, the unit would be useful and informative†, although it would not necessarily displace all special nomenclature.

In the opinion of the writer, however, the position in acoustics has become such that the introduction of a purely mathematical logarithmic unit like the brig is urgently necessary. That a logarithmic terminology is needed in acoustics (the ear dealing comfortably with over a million-millionfold range of audible energy) has long been recognized. At least two units, the T.U. and the S.U., have been tried and abandoned as unsatisfactory. A unit called the "bel" is at present in vogue for representing a tenfold change of acoustical power but, possibly owing to acoustical associations of its name, it has come to be used also in a slightly different sense to represent the results of aural balancing of sounds, referred to later. Consequently many find difficulty in regarding it as a simple logarithmic unit even when it is used as such, and some suspect that it involves assumptions about the universal

* See A. H. Davis, *Discussion on Audition*, p. 82 (*Phys. Soc.* 1931).

† A change of 1 stellar magnitude is a change of 4 decibrigs in photometric intensity; a unit change of optical density in a photographic negative is a change of optical opacity of 1 brig; a vibrating system having a logarithmic decrement of δ would exhibit a decay in amplitude of vibration of $0{\cdot}43\delta$ brigs per cycle; radium decays in mass at a rate of about 1·8 centibrigs per century, or 5·5 micro-micro-brigs per sec.

validity of the Weber-Fechner law. In short, a position has arisen where precise statements are cumbersome, laxity is general, and confusion is common. The difficulty would be overcome if the brig were adopted for general use. If then a special unit, such as the bel, were required for any particular purpose in acoustics, it would be defined simply as a change of 1 brig in acoustical intensity. All readers would then understand that it was simply a term for convenient logarithmic expression of results, and the writer suggests that the term bel should be used as defined above, if at all.

It would be legitimate to apply the brig to express the extent to which one sound differed in intensity from a sound of unit intensity, thus "n decibrigs above unit intensity"; or from a sound of minimum audible intensity, thus "m decibrigs above threshold intensity."

A new term should, however, be adopted for use in aural measurements of the loudness of sounds, such as those where the loudness of a noise is measured aurally by observing the intensity-level of a note of standard pitch, say 1000 c. sec., which has been adjusted in strength until an observer has become equally aware of the noise and of the note.

In the past the bel, or more particularly the decibel, has been employed in expressing loudness-levels as well as intensity-levels. The practice has led to confusion and the writer tentatively suggests the adoption of, say, a "phon" defined* somewhat on the following lines: When an observer, listening simultaneously to a noise and to a note of chosen standard pitch having an intensity n decibrigs or decibels above the accepted threshold, judges that he is equally aware of the two sounds, the noise is said to have a loudness of n phons.

It will be realized that the two forms of expression for intensity and the term "loudness" are all useful. The first two express the intensity of a sound in brigs, but above different zeros. The third expresses the intensity of a comparison note which is as loud as the sound. Since the term *brig* or *bel* can be applied to the energy (intensity) of the noise, a different term *phon* has been chosen to express loudness, where in fact the energy of the noise is quite undetermined, and only the energy of a comparison note is expressed.

It is convenient to compare the above proposals with present practice, which appears to be to omit to mention the units and the zero when using decibels and to say:

an intensity-level of n decibels;
a sensation-level of n decibels;
a loudness of n decibels.

In general these levels are all different, and an increase of intensity-level by 10 decibels does not necessarily increase the loudness by 10 decibels: indeed the increase in loudness may even be 20 decibels. Consequently confusion arises as to the meaning and limitations of the decibel.

* As defined above, the phon is approximately the smallest loudness-change ordinarily detectable by the ear. The phon is in fact the German equivalent of the decibel. The present proposal is to use the word *decibrig* or *decibel* for intensity relations and *phon* for expressing loudness.

The writer, in his own published noise-measurements, has attempted to avoid the difficulty by more explicit forms of expression. He has for instance made it clear in his tables of noise-measurements that the decibels mentioned relate to the comparison note, not to the noise which was judged to be equally loud. The form of expression is almost impracticably cumbersome, however, and readers who have no time for elaborate niceties are apt to transfer the decibels to the noise without further ado.

If the proposals of the present paper were adopted the position would be distinctly clarified. One would say:

an intensity of n decibrigs or decibels above unit intensity;

an intensity of n decibrigs or decibels above threshold intensity;

a loudness of n phons.

The nomenclature proposed would be consistent with the fact that an increase of 10 decibrigs or decibels in the intensity of a sound, as measured above unit intensity, would necessarily be accompanied by an increase of 10 decibrigs or decibels as measured above threshold. At the same time it would separate questions of intensity from questions of loudness, and would save a reader from the error of thinking that an increase of 10 decibrigs or decibels in intensity is necessarily associated with a 10-phon increase in loudness.

It is clear from the above that the essential requirements in acoustics are the quantitative relations which have been named the brig and the phon, one purely logarithmic and the other the basis of an aural scale. If the brig were adopted as a general mathematical unit, there would be no logical need for a special acoustical unit (bel) applicable to intensity and power. Its retention would depend upon the balance of convenience.

The writer suggests that the brig has a strong claim to adoption as a general mathematical unit for meeting the need, already acutely felt in acoustics, for a purely logarithmic form of expression.

535·338·3:546·295

A NOTE ON THE HYPERFINE STRUCTURE IN THE ARC SPECTRUM OF XENON

By E. GWYNNE JONES, Ph.D., Beit Research Fellow,
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Communicated by Prof. A. Fowler, F.R.S., March 23, 1934. *Read June* 1, 1934.

ABSTRACT. The hyperfine structures of the Xe-I lines λλ 9045, 9799 and 9923 are described and analysed, and the hyperfine separations of the terms $2p_9$ and $2p_{10}$ are derived. It is also found that the lines $1s_5$-$2p$ are readily self-reversed. Previous nuclear spin data are confirmed.

§ 1. INTRODUCTION

The hyperfine structures occurring in the arc spectrum of xenon, from λ 4400 to λ 8800, were described and analysed in an earlier paper by the present writer*. A new type of infra-red sensitive emulsion† has since made it possible to extend these observations into the region λ 9000 to λ 10,000, which contains the important Xe-I lines λ 9045 ($1s_5$ $2p_9$), λ 9799 ($1s_5$ $2p_{10}$) and λ 9923 ($1s_4$–$2p_9$). The analysis of the new line structures leads to the hyperfine structures of the terms $2p_9$ and $2p_{10}$.

§ 2. EXPERIMENTAL

As in the previous work, the xenon lines were excited in a Geissler tube (used in the side-on position with currents up to 5½ mA.) and examined by means of Fabry-Pérot etalons mounted in a glass spectrograph. The photographs were all taken on Eastman I.Q (Xenocyanine) plates, exposure times of 3 to 9 hours being necessary.

§ 3. DISCUSSION OF RESULTS

The observed hyperfine structures of the lines λλ 9799 and λλ 9923 are shown in figures 1(*a*) and 2(*a*), in which the estimated intensities are represented by the heights of the appropriate lines, the positions being given in units of 10^{-3} cm.$^{-1}$ The structure of λ 9799 is very similar to that of the next series member, λ 4792, but the resolution of the interferometer is so much greater in the infra-red that, in λ 9799, the components *a* and *B* are completely separated, figure 1(*a*) and plate (*c*). The structure of λ 9923 is like that of λ 4734 but on a much smaller scale, so that *a* and *B* now fall together, figure 2(*a*). In the case of λ 9045, which is considerably less intense than the other lines, only the three strongest components were observed

* E. G. Jones, *Proc. R.S.* **144**, 587 (1934).
† C. K. Mees, *J. Opt. Soc. Am.* **23**, 232 (1933).

(roughly $-0{\cdot}04$, 0 and $+0{\cdot}04$ cm.$^{-1}$). These components are sufficient to provide a check on the $2p_9$ term structure.

In agreement with the previous analysis, these line structures may be explained by assigning the nuclear moments $I = 0$ to the even isotopes, $I = \frac{1}{2}$ to Xe_{129} and

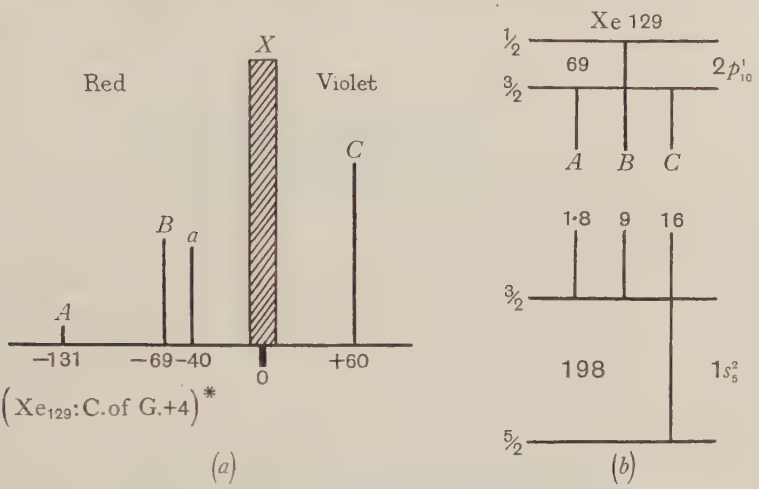


Figure 1. λ 9799 ($1s_5^2$–$2p_{10}^1$).

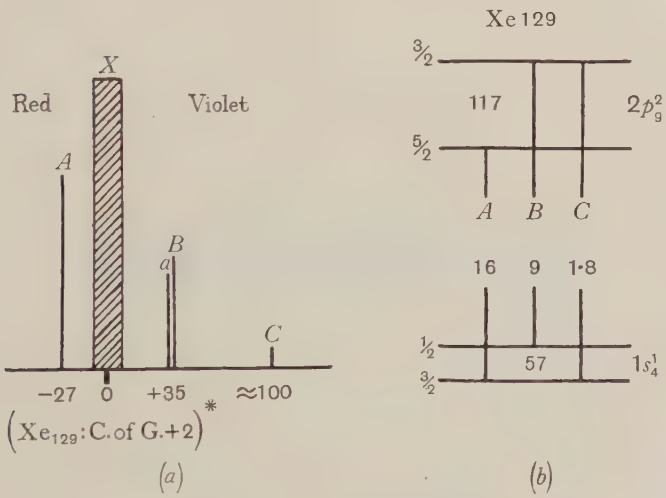


Figure 2. λ 9923 ($1s_4^1$–$2p_5^0$).

$I \geqslant 3/2$ to Xe_{131}. The complete analysis for Xe_{129} is given in figures 1(*b*) and 2(*b*). In figures 1(*a*) and 2(*a*) the components assigned to Xe_{129} are indicated by capitals and those to Xe_{131} by small letters. Since each of the lines represents a transition from an upper level ($2p_9$ or $2p_{10}$) of previously unknown structure to one ($1s_4$ or $1s_5$)

* The central component X is quite sharp and coincides with the centre of gravity of the components assigned to Xe_{129}.

of known structure, the hyperfine structures of the terms $2p_9$ and $2p_{10}$ may now be uniquely determined from the new data. The values thus obtained are shown in table 1.

Table 1

Term	J-value	Hyperfine separation of Xe_{129} ($cm.^{-1}$) where $I=\frac{1}{2}$
$2p_9$	2	−0·117
$2p_{10}$	1	−0·069

§ 4. SELF-REVERSAL OF THE LINES

It has been found that all the lines $1s_5$–$2p$ ($\lambda\lambda$ 8231, 8409, 8819, 9045 and 9799) are very easily reversed. With currents as low as $\frac{1}{2}$ mA., all these lines are completely reversed when the tube is viewed end-on, but with the side-on arrangement no trace of self-absorption is found, even at $7\frac{1}{2}$ mA. It would thus appear that these lines should be used only with the greatest caution as comparison lines in grating spectra.

§ 5. DESCRIPTION OF PLATE

The hyperfine structure of the line λ 9799 is shown in the plate by means of three sets of etalon patterns. The resolution of the instrument increases with increased etalon separation, so that each pattern shows more detail than the preceeding one. In plate (*c*) the dispersion is approximately 1·65 Å./mm.

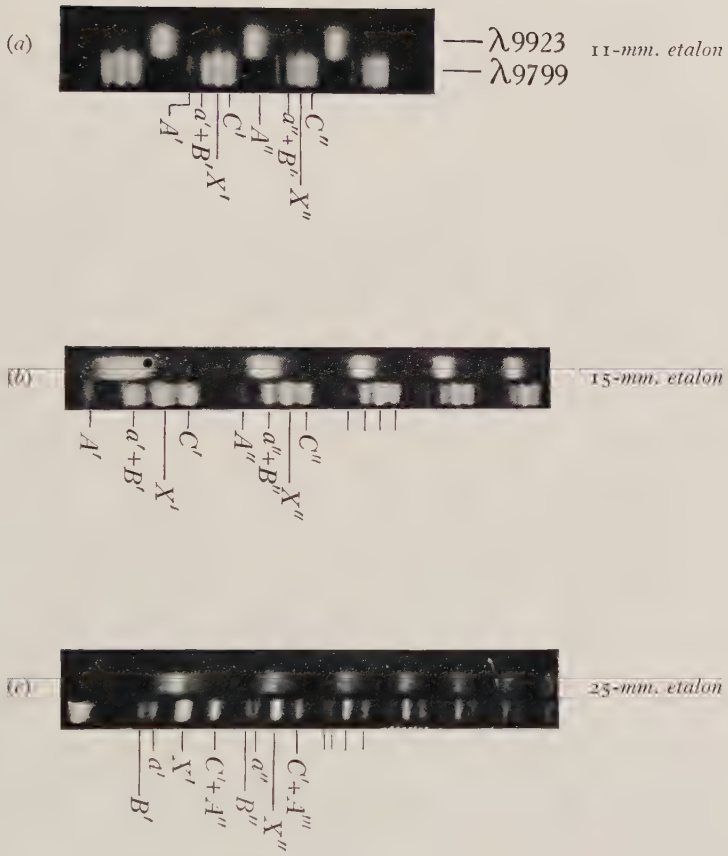
§ 6. ACKNOWLEDGMENT

The writer wishes to express his thanks to Prof. A. Fowler, F.R.S., in whose laboratories the work was done, for his kindly interest.

DISCUSSION

Lord Rayleigh said that the photographs were beautifully clean. Had the author had any trouble with the xenocyanine plates which he used? Some friends of the speaker had used them but had not obtained satisfactory results. It would be interesting to know how far into the infra-red a photographer could go. There was a tradition at the Imperial College that Sir William Abney had photographed a boiling kettle in the dark, but the photograph had never been on view. It was said that the Agfa Co. had produced plates allowing of a great extension of the solar spectrum in the infra-red.

Author's reply. The reproductions do not show that the plates have a rather dense general fog which considerably increases the density of the negative, thus making it difficult to obtain a good print. This is not a great disadvantage in spectroscopic work but would restrict snapshotting activities to shorter wavelengths. It is, however, only fair to point out that the makers list the plates as "spectroscopic". The chief difficulty seems to lie in the rapid decrease in sensitivity during the first hour after the ammonia sensitizing. The Eastman plates go farther into the infra-red than any Agfa plates that I have used.



Etalon patterns of λ 9799 (see figure 1 *a*).

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A DETERMINATION OF THE ELECTRICAL CONSTANTS OF THE EARTH'S SURFACE AT WAVE-LENGTHS OF 1·5 AND 0·46 M.

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Communicated by Dr R. L. Smith-Rose, March 28, 1934. *Read June* 15, 1934.

ABSTRACT. A description is given of experiments made to determine the reflection coefficient and the electrical constants of samples of the earth's surface for radiation having wave-lengths of 1·5 and 0·46 m. It is shown that the most sensitive condition for finding the electrical constants of a substance from a study of its reflecting properties for electromagnetic waves is obtained when the radiation is incident normally on the reflector. The experiments described show that in this case the reflection coefficient of copper gauze is practically unity at both wave-lengths. There appears to be little difference in the reflecting properties of ordinary soil and soil covered by grass. It is suggested that the reason for this result may be that in both cases the bulk of the reflection takes place at a small distance below the surface. For radiation having a wave-length of 1·5 m. the dielectric constant of soil lies between 7 and 16 and the conductivity below about 10×10^8 e.s.u. This upper limit to the value of the conductivity is lower than that, 95×10^8 e.s.u., found previously on the same site at a wave-length of 1·6 m. Part of this discrepancy may be accounted for by the dryness of the season during which the present experiments were made. The results at a wave-length of 0·46 m. indicate possible values of between 7 and 20 for the dielectric constant and up to 40×10^8 e.s.u. for the conductivity of the soil.

§ 1. INTRODUCTION

Much interest has been taken in the last few years in the production of electromagnetic radiation at wave-lengths shorter than 3 m., corresponding to frequencies above about 10^8 c./sec. Very little attention, however, has been given to the propagation of the radiation produced. This propagation is intimately bound up with the electrical properties of any bodies such as the earth's surface or buildings present in the path of the waves. About the latter end of the nineteenth century many physicists made measurements on the electrical properties of different materials at these high frequencies, but the substances tested were not those which would affect the use of the waves for communication; moreover the experimenters had to be content with generators capable of producing only damped oscillations. With the modern three-electrode valve used as a Barkhausen-Kurz oscillator it is possible to obtain sustained oscillations of very high frequency. The present paper describes a series of experiments made at wave-lengths of 1·5 and 0·46 m. (frequencies of 2×10^8 and $6{\cdot}5 \times 10^8$ c./sec.), in order to determine the

reflecting properties of typical samples of the earth's surface for radiation of these frequencies. In addition to being of some practical importance in determining the effect of the earth's surface on the propagation of waves belonging to this part of the spectrum, the type of experiment described can be made to yield valuable data on the characteristics of electromagnetic radiation close to a reflecting surface, represented in this case by the ground. The general study of propagation near reflecting surfaces will be pursued at a later date.

§ 2. PREVIOUS WORK

Smith-Rose and the author[1, 2] have made an experimental study of the electrical properties of the earth's surface for radiation at a series of wave-lengths between 10 and 1·6 m. In these experiments two waves from a local transmitter arrived at the receiver, one directly and the other after reflection at the ground. The field-strength at the receiver was measured for different distances between the transmitter and receiver while their respective heights above the ground remained constant. In this way a variation was obtained in the angle of incidence on the earth's surface of the ray reflected from the transmitter to the receiver by the ground. The observations were sufficient to determine approximately the coefficient of reflection at the earth's surface for this ray for a series of values of the angle of incidence. The electrical constants of the earth's surface for radiation having a frequency equal to that of the transmitter were then deduced from an application of Fresnel's equations for the reflection of electromagnetic waves by an imperfect conductor. The general result of this research was that over the range of wave-lengths observed the conductivity of the ground appeared to lie in the region of 10×10^8 e.s.u., corresponding to a resistivity of 900Ω.-cm., for a typical field site for which experiments made at longer wave-lengths (about 300 m.) indicated a value of about 10^8 e.s.u. The dielectric constant could not be determined from the experiments and was assumed in the analysis to have a value of 10. Strutt[3] has described similar experiments made on a wave-length of 1·42 m., from the analysis of which he was led to the same result—that the conductivity of the ground appeared to increase with increase in frequency of the incident waves.

§ 3. DESCRIPTION OF TRANSMITTERS AND RECEIVERS

Two separate transmitters were used for the two frequencies investigated. Both were of the type known as Barkhausen-Kurz oscillators, and as these are not very well known a short description of them is given. It is difficult to obtain oscillations from an ordinary valve circuit at wave-lengths shorter than about 2 m. It has been found, however, that oscillations of much shorter wave-length can be obtained if the valve used has cylindrical electrodes and if a high positive potential with respect to the filament is applied to the grid and a small positive or negative potential to the anode: in fact, a Barkhausen-Kurz valve oscillator, as such a transmitter is usually called, is somewhat similar to an ordinary retroactive valve-

oscillator in which the potentials applied to the grid and anode have been interchanged. There is one fundamental difference, however, which is that in a valve oscillator using retroaction the wave-length of the oscillations produced is controlled by the constants of the external circuit, whereas in the Barkhausen-Kurz type the wave-length is primarily dependent on the time of transit of the electrons between the electrodes of the valve and therefore on their dimensions and potentials. This feature was utilized in the experiments described below in the following way; two valves having different electrode-dimensions were chosen so that the same battery supply could be used to obtain radiation of two different frequencies.

The valve used in the higher-frequency oscillator was of the AT40 type. Small variable condensers were placed between the grid and the two filament pins of the valve. These condensers were formed by fixing a small brass washer to the grid pin and similar washers on spindles to the filament pins. The latter could be rotated about the filament pins as axes so as to vary the terminal capacity between the grid and filament leads The aerial was formed by a copper-rod extension of the anode pin in line with the anode support within the valve. With this arrangement strong oscillations were obtained at a wave-length of 0·46 m., when the grid-potential and current were maintained at 300 V. and 150 mA. respectively. A preliminary experiment showed that most of the radiation from the transmitter took place from the anode support and extension, which are considered in the analysis below to constitute the radiating source.

The longer-wave oscillator incorporated a valve of type ES 250. The anode in this valve was extended externally in the same way as in the oscillator described above, but in this case a small dry battery was fixed near the valve by means of which the anode was given a small negative potential with respect to the negative end of the filament. No tuning-condensers were used, because strong oscillations at a wave-length of 1·5 m. were obtained without them when the grid-voltage was 300 and the grid-current remained in the region of 220 mA.

The operating conditions of a Barkhausen-Kurz oscillator are so critical that unless they are maintained within narrow limits the oscillations cease. This feature was found to be very useful in the following experiments, for whether the transmitter was used continuously or intermittently the output remained constant to 1 or 2 per cent so long as the valve was giving rise to oscillations.

The receiver in each experiment consisted simply of a copper rod forming, with a 10-mA. non-contact thermojunction at its mid-point, a half-wave-length aerial. A d.-c. calibration of the thermojunction showed that the current output was very nearly proportional to the square of the heater current. In the analysis of the results this proportionality is assumed to be accurate at the high frequencies used.

§ 4. EXPERIMENTAL PROCEDURE

In the earlier experiments the transmitting and receiving aerials were arranged parallel to one another and a fixed distance apart in the same horizontal plane, figure 1. The output at the receiver was observed for different heights of this plane above the ground. Figure 2 gives a graphical illustration of the type of result obtained; it relates the reading of the microammeter connected to the thermo-junction at the receiver with the height of the plane containing the transmitter and receiver. It will be seen that as the height of this plane is altered the current at the receiver passes through a series of maximum and minimum values. The reason for this can readily be understood from a study of figure 1. In this figure T and R *h* represent respectively the transmitter and receiver at a common height h above the ground. Radiation from T reaches R in two ways, one along the direct path

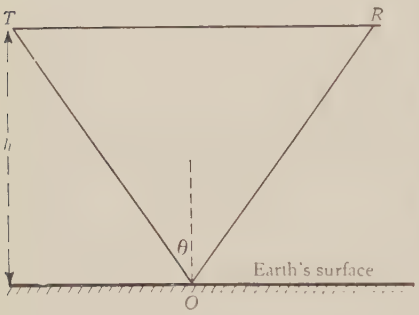

Figure 1.

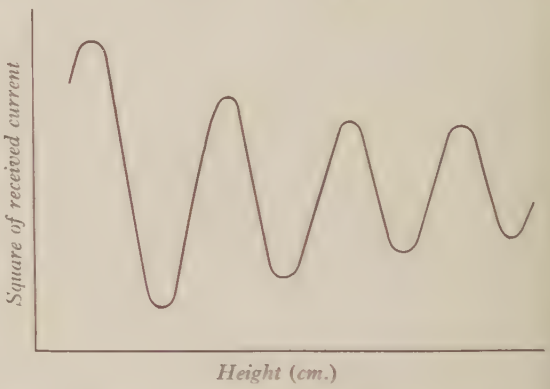

Figure 2. Graph illustrating relation between square of current in the receiving aerial and height above the ground.

TR and the other along the path TOR after reflection from the ground at O. The resultant magnitude of the field at R depends upon the amplitudes of these constituent waves and also on their phase-difference at R. This phase-difference depends on the actual path-difference between TR and TOR and also on the phase-change introduced on the ray TOR on reflection at the ground. The intensity of the resultant field at the receiver passes through maximum and minimum values respectively when the two waves at R are approximately in the same and in opposite phase. The difference in path between TR and TOR varies with change in the height h so that an interference pattern is obtained in the current received at R in the way shown in figure 2. The amplitude of the variation of the field-strength at the receiver decreases as the height h increases, for the effect of the reflected ray becomes small compared with the direct ray when the length of the path TOR becomes large compared with that of TR.

The phase-difference between the two waves at R is not merely that due to the difference in length of the two paths, for a phase-change is introduced in the ray TOR on reflection from the ground at O. This change of phase is dependent

on the electrical constants of the ground near O and also on the angle of incidence θ of the ray on the earth's surface. The amplitude of any wave reflected by the earth's surface is also determined by the constants of that surface and by the angle of incidence, the reflection coefficient increasing in general with increase in the value of the dielectric constant or conductivity of the reflecting substance. The exact positions of the peaks and dips as shown in figure 2 and, more particularly, their relative magnitudes are determined, therefore, by the reflection coefficient of the ground for radiation of the frequency used in the experiment. As the distance TR remains constant the angle of incidence θ changes with alteration of the height h. The electrical constants of the earth's surface are determined from such an interference pattern as figure 2 by finding those values from which the most closely similar curve can be obtained by theoretical analysis. This entails much tedious computation, for the reflection coefficient must be calculated for a sufficient number of values of the angle of incidence θ to determine accurately the positions and relative magnitudes of successive maxima and minima. The computation is much simplified if the angle θ remains constant as the height h is altered, since the reflection coefficient then has only one value. It can be shown theoretically that for a given change in the electrical constants of a reflecting surface the maximum change in the reflection coefficient occurs for normal incidence, that is when the angle θ, figure 1, is zero. The greatest sensitivity is obtained, therefore, in the determination of the electrical constants of the earth's surface when one aerial is arranged vertically above and parallel to the other. The amount of computation is then greatly reduced, since the formula for the reflection coefficient becomes relatively simple in the case of normal incidence and also its value remains the same for all heights of the transmitter and receiver.

The experiments were repeated, therefore, with the two aerials arranged horizontally and the transmitting aerial vertically above the receiver. Each aerial was fixed at the end of a horizontal beam and the two beams were attached to a vertical pole so as to remain at a constant distance apart. The lengths of the two beams and the distances between them were respectively 150 cm. and 250 cm. for the experiments on a wave-length of 1·5 m., and 50 cm. and 150 cm. when the wave-length was 0·46 m. The transmitting and receiving system could be moved as a unit up and down the vertical pole which had holes 5 cm. apart drilled in it. By means of a peg fitted beneath the lower beam to which the receiving aerial was connected, one observer could alter the height of the system and at the same time preserve a constant distance between the transmitter and receiver. The supply leads to the transmitting valve were twisted round one another and fixed to the wooden beam supporting the transmitter. They were supported on a post at some distance from the site of the experiment so that they remained sensibly horizontal and perpendicular to the transmitting aerial for all heights of the latter. No chokes were inserted in the supply leads, for the radiation from them to the receiver was found to be negligible. The microammeter at the receiver was placed at the vertical-pole end of the beam supporting the receiving aerial and was connected to the thermojunction by thin twisted copper leads.

The apparatus was set up so that the plane containing the transmitting and receiving aerials was vertically above the medium whose electrical properties were under investigation. The reading of the microammeter at the receiver was observed by telescope for different aerial-heights which were altered in steps of 5 cm. There is some doubt about the applicability of Fresnel's equations for points very near a reflecting surface and most of the observations were therefore made with the receiving aerials at heights of more than one wave-length above the ground. Interference patterns similar to that given in figure 2 were obtained. The reason for these is the same as that given above, except that in the present case the phase-change introduced on reflection at the ground remains constant since the angle of incidence is zero throughout the experiment.

§ 5. METHOD OF ANALYSIS

The conditions for maximum and minimum currents in the receiving aerial occur respectively when the two waves at the receiver are approximately in the same and in opposite phase. The coefficient of reflection at the earth's surface is
R complex; suppose its modulus to be of magnitude R. If the fixed distance between
d the transmitter and receiver is d and the observed height of the receiving aerial for maximum or minimum current is h then the measured current at this position should be proportional to

$$\left(\frac{1}{d} \pm \frac{R}{d + 2h}\right),$$

the positive or negative sign depending on whether the current observed is a maximum or a minimum.

The experimental results were plotted graphically in the form shown in figure 2 and the heights and amplitudes of the receiver-current maxima and minima were carefully determined. By adding the relations given by the above proportionality for the maximum-current conditions and repeating the procedure with the minimum, an average value of R was determined. Each series of experimental results was checked by determining the value of R obtained by means of different groups of maxima and minima. In this way any doubtful peaks and troughs could be eliminated when the average value of R was being found from the complete curve. The agreement between the different values of R was surprisingly good and usually sufficient to determine R with some certainty to the second significant figure.

The reflection coefficient, as has been mentioned before, is complex, and the above analysis determines its modulus only and not its two rectangular components.
K, K' If the complex coefficient of reflection R be represented by $K + jK'$, the values of K and K' are deducible from Fresnel's equations for the reflection of electromagnetic waves at a conducting surface. In the case under consideration in which plane-polarized radiation is incident normally on the reflector the values of K and K' are given by the relations

$$K = \frac{1 - \sqrt{(\kappa^2 + 4\sigma^2/f^2)}}{1 + \sqrt{(\kappa^2 + 4\sigma^2/f^2)} + \sqrt{[2\{\kappa + \sqrt{(\kappa^2 + 4\sigma^2/f^2)}\}]}} \quad \ldots\ldots(1),$$

and

$$K' = \frac{\sqrt{[2\{-\kappa + \sqrt{(\kappa^2 + 4\sigma^2/f^2)}\}]}}{1 + \sqrt{(\kappa^2 + 4\sigma^2/f^2)} + \sqrt{[2\{\kappa + \sqrt{(\kappa^2 + 4\sigma^2/f^2)}\}]}} \quad \ldots\ldots(2),$$

in which κ and σ are respectively the dielectric constant and conductivity in electrostatic units of the reflecting surface for radiation of frequency f. When the conductivity (or the dielectric constant) of the reflector is infinitely great the component K' is zero and K becomes equal to -1. This means that the wave undergoes a change in phase of 180° on reflection. When the reflector is an imperfect conductor the values of K and K' are negative and positive respectively. The advance of phase on reflection, equal to $\tan^{-1}(K'/K)$, is less than 180° and the amount of the change is a measure of the ratio K'/K. The magnitude of $\tan^{-1}(K'/K)$ was obtained in the present experiments by observing the displacement of the positions of maximum current from those deduced theoretically for the case of a perfect reflector.

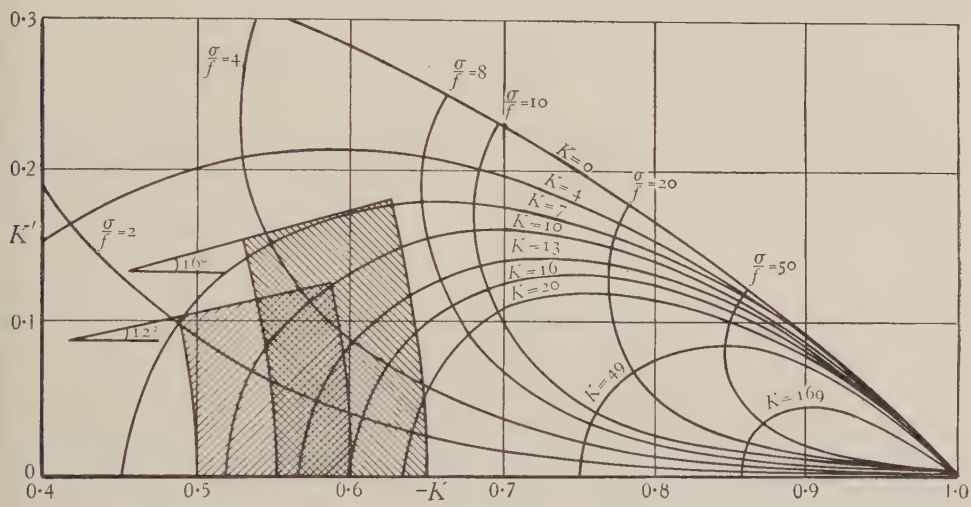

Figure 3. Curves giving the in-phase and in-quadrature components of the coefficient of reflection at zero angle of incidence for a medium having given electrical constants.

A determination of the magnitudes of the modulus of the reflection coefficient and the ratio of its two rectangular components is sufficient to determine the electrical constants of the reflecting surface under investigation. The two series of intersecting curves in figure 3 are obtained from relations (1) and (2) above by in turn assuming κ and σ/f as the constant and parameter, and *vice versa*, in these expressions. The point of intersection of any two of these curves determines the values of both K and K' for the common values of κ and σ/f for the two curves. In the same way corresponding values of κ and σ/f can be assigned to any point in the diagram. A knowledge of the modulus of the reflection coefficient and the value of $\tan^{-1}(K'/K)$ determines both the length of the radius vector from the origin of coordinates to a point and the angle this vector makes with the axis of abscissae. The position of the point is thus specified and, therefore, the values of both κ and σ/f for the reflector can be given. In any experiment there is always a certain limit to the accuracy of determination of both the reflection coefficient and

its components, so that the point required in the diagram degenerates into a small area which defines limits to the possible values of the electrical constants of the reflector. As has been mentioned above, the magnitude of K is negative. For convenience, however, the axes of coordinates in figure 3 have been arranged so that in this diagram an apparently small positive value for $\tan^{-1}(K'/K)$ represents actually an advance of phase on reflection of $180° - \tan^{-1}(K'/K)$.

§ 6. DISCUSSION OF RESULTS

Determination of the modulus of the reflection coefficient for a wave-length of 1·5 *m.* Experiments were made on various typical substances. The analyses were performed in the way outlined above and the results are given in table 1.

Table 1

Date	Surface		Reflection coefficient
	Nature	Dimensions (m.)	
24. 7. 33	Grass	Field site	0·52
4. 7. 33	Soil	5 × 5	0·55
7. 7. 33	,,	,,	0·52
29. 8. 33	,,	,,	0·52
4. 7. 33	Iron wire netting of $\frac{1}{2}$-in. mesh	1·8 × 4·0	0·98
4. 7. 33	Copper gauze	2·0 × 4·0	1·0

There appears from this table to be little difference between the reflecting properties of soil and those of ground covered by short grass. The reason for this may be that in each case the layer at which most of the reflection takes place is at a small distance below the earth's surface, but the experiments were not sufficiently accurate to determine whether this was actually the case. The small difference between the reflection coefficient of copper gauze and wire netting would indicate that although the use of a wire-netting screen under a transmitting aerial might be advisable at this high frequency, little extra gain in efficiency would be obtained with the much more costly copper screen. As the reflection coefficient of a given material usually increases with a decrease in frequency, wire netting should approximate even more closely to a perfect reflector for wave-lengths greater than 1·5 m. At long wave-lengths, however, the reflection coefficient of the earth becomes comparable with unity and little would then be gained by using either a copper or an iron wire-netting screen below the transmitting aerial.

Determination of the two components of the reflection coefficient for a wave-length of 1·5 *m.* The two components of the reflection are determined from the differences between the actual heights of the maximum- and minimum-current positions and the heights deduced analytically when the earth is assumed to be a perfect reflector. It was observed that although the amplitude of the reflected wave indicated that both copper gauze and iron wire netting were substantially perfect reflectors for the wave-length of 1·5 m., the positions of the peaks and dips of received current

did not correspond exactly with those found theoretically for a perfect reflector. Relations (1) and (2) above show that the values of K and K' are negative and positive respectively. This means that the retardation of phase on reflection must always be greater than 180°, and therefore that for an imperfect reflector the heights for maximum and minimum currents should be lower than those calculated for a perfect reflector in which the change of phase is 180°. There appears to be no satisfactory reason to account for the increase in heights observed in the case of the copper-gauze and wire-netting experiments. It may be due to the fact that the reradiation from the earth's surface is not quite plane at the receiving aerial. This would account for the direction of the displacement, but if it were the sole reason the discrepancy in height should decrease as the receiving aerial is placed at increasing distances from the ground. This result was not observed. Another possible but improbable explanation is that both screens had a capacitive reactance at the frequency in question.

When the results for soil and grass-covered soil were analysed the displacement of the peaks and troughs was found to be in the correct direction. There was little difference between the results for these two substances, the average displacement having a magnitude of about 3 cm. The unaccountable displacement in the case of the copper-gauze and wire-netting experiments was less than 2 cm. Assuming this latter displacement to be due to some propagation effect, then the maximum possible displacement for soil would be 5 cm. This latter value gives an upper limit to the phase-change on reflection and therefore to the value of the angle $\tan^{-1}(K'/K)$. A displacement of 5 cm. corresponds on a wave-length of 1·5 m. to a value of 168° for $\tan^{-1}(K'/K)$. All possible values of $\tan^{-1}(K'/K)$ for soil or grass, therefore, must lie between 180° and 168°. Suppose the accuracy of the experiment to be such that the reflection coefficient $\sqrt{(K^2 + K'^2)}$ for the soil and grass cannot be determined within closer limits than 0·5 to 0·6. All possible combinations of the values of K and K' must then lie within the sector of an annulus shown hatched in figure 3 and having an angle of 12° while the radii of the two circular sections are 0·5 and 0·6. It will be seen that with the limits given above the value of the dielectric constant of soil or grass must lie between 7 and 16 for radiation on a wave-length of 1·5 m. The upper limit to the value σ/f for the same substance is approximately 5. Figure 4 shows a comparison between the experimental results obtained for a grass-covered site and those obtained theoretically on the assumption that the values of κ and σ/f are 10 and 3 respectively, these values being approximately the mean of those possible according to the above analysis.

The dielectric constant of the ground is usually assumed to be 10 for long-wave radiation where its value is not critically important in determining the propagation of the waves. It is interesting to note that the mean value of the two limits given above is not far removed from this value. The value of 5 for σ/f corresponds for a wave-length of 1·5 m. to a value of 10×10^8 e.s.u. for the conductivity of soil*. Smith-Rose and the author[1] gave a value of 95×10^8 e.s.u. as the conductivity of

* A conductivity of 10^8 e.s.u. corresponds to a resistivity of 9000 Ω.-cm.

soil for a wave-length of 1·6 m. The present type of experiment is certainly more accurate than that from which this high value was determined: also the present experiment gives 10×10^8 e.s.u. as the upper limit to the value of the conductivity. The actual value, therefore, is probably less than that previously determined. Part of the discrepancy may be explained by the fact that the present experiments were made during an exceptionally dry season.

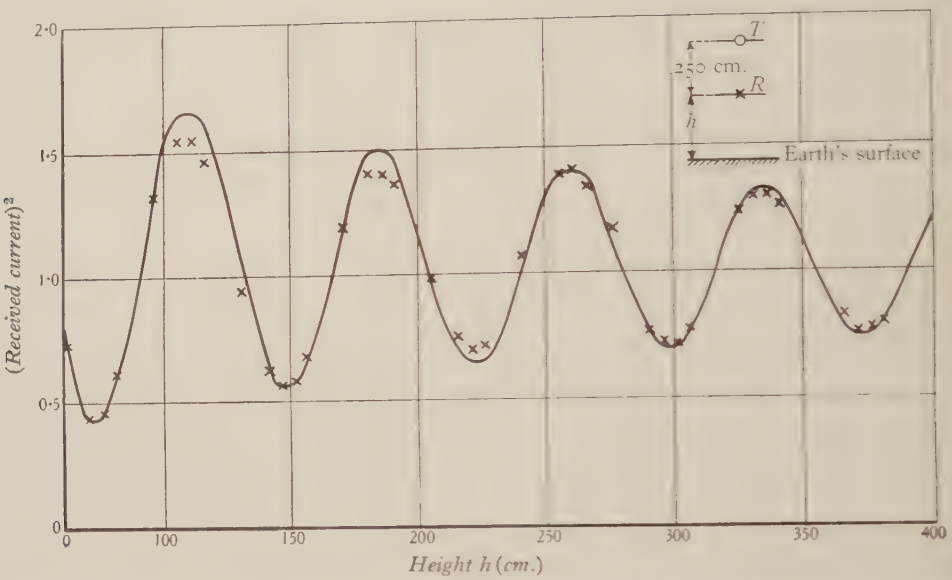

Figure 4. Relation between square of current in receiving aerial and its height above the ground.
× × Experimental values obtained July 27, 1933, grass, $\lambda = 1{\cdot}5$ m.
—— Theoretical curve calculated for $\kappa = 10$, $\sigma/f = 3$.

Determination of modulus of reflection coefficient for a wave-length of 0·46 *m.* Experiments carried out with a wave-length of 0·46 m. on the same sites as those detailed in table 1 yielded the results given in table 2.

Table 2

Date	Surface		Reflection coefficient
	Nature	Dimensions (m.)	
12. 5. 33	Soil	5 × 5	0·6
12. 5. 33	Copper gauze	2·0 × 4·0	0·94

The value of R for soil appears to be greater than that obtained for the wave-length of 1·5 m. This increase may be due to the fact that the experiments on 0·46 m. were made before the very dry period during which the experiments on 1·5 m. were carried out.

Determination of the two components of the reflection coefficient for a wave-length of 0·46 *m.* The determination of the two components of the reflection coefficient

of soil is more difficult for 0·46 m. than for the longer wave-length. This is due to the decrease in the accuracy of determination of the upper limit to $\tan^{-1}(K'/K)$. The average displacement of the positions of the peaks of current in the case of soil from those found for the copper gauze was approximately 2 cm., which gives a value of 164° for $\tan^{-1}(K'/K)$. If the reflection coefficient for soil is assumed from the experiments to lie between 0·55 and 0·65, the annular sector obtained in figure 3 is that given by the larger hatched area. It will be seen from this figure that the dielectric constant of soil for radiation of wave-length 0·46 m. then lies between 7 and 20. In the same way the upper limit for σ/f is seen to be 7. This corresponds to a value for the conductivity of approximately 40×10^8 e.s.u.

§ 7. CONCLUSIONS

The results of the series of experiments described above may now be summarized. The reflection coefficients of both a copper-mesh screen and half-inch-mesh iron netting appear to be approximately unity for radiation of wave-length of 1·5 m., and the reflection coefficient of copper gauze is still very high at 0·46 m. The experiments on a wave-length of 1·5 m. gave approximately the same value for the reflection coefficient of bare soil and that of soil covered by grass. It is to be assumed from this result that the bulk of the reflection in either case must take place in the soil itself. The dielectric constant of the soil for this wave-length appeared to lie between 7 and 16 and the conductivity below 10×10^8 e.s.u. The similar results on 0·46 m. indicate possible values between 7 and 20 for the dielectric constant and below 40×10^8 e.s.u. for the conductivity.

§ 8. ACKNOWLEDGMENTS

The work described in this paper has been carried out as part of the programme of the Radio Research Board and acknowledgment is due to the Department of Scientific and Industrial Research for granting permission for publication. The author is also indebted to Dr R. L. Smith-Rose for advice on the method of presentation of the paper, and to Messrs A. C. Haxton and H. M. Bristow for assistance in the experimental and computing work involved.

APPENDIX

Consideration of the difference between reflection at a perfect conductor and at a perfect dielectric

A study of figure 3 will show that when either the dielectric constant or the conductivity of a medium becomes great the value of K'/K approaches zero. In other words the phase-change $\tan^{-1}(K'/K)$ on reflection at normal incidence from such a medium is approximately 180° and, therefore, no observations made outside the medium under these conditions can determine absolutely whether the medium has a large conductivity or a large dielectric constant. The reason for this apparently

anomalous result is readily understood from a consideration of the boundary conditions. If the electric field at the surface due to the incident wave is E_1 in the first medium and E_2 in the second, denser, medium, then with the symbols given above

$$\{(1 + K)^2 + K'^2\}^{\frac{1}{2}} E_1 = (\kappa - 2j\sigma/n) E_2.$$

The right-hand side of this expression reduces to kE_2 when the second medium has a high dielectric constant, and to $-2j(\sigma n) E_2$ when it has a large conductivity.

It will be seen that although the resultant field just outside the medium is the same in each case, the refracted waves differ in phase by 90°. No observations made with reflection methods at normal incidence and external to such a medium can determine, therefore, absolutely whether it has a high conductivity or a large dielectric constant. When reflection is obtained at angles of incidence other than zero the type of medium can be determined owing to the tangential boundary conditions. Maximum sensitivity would appear to be obtainable, however, if the wave within the medium could be observed. This suggests that when the reflection coefficient is high a laboratory method of investigation should be adopted so that the resistance and reactance of a given sample of the material may be found. From a knowledge of these two quantities the values of the dielectric constant and conductivity can readily be determined.

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THE MEASUREMENT OF THE ELECTRICAL CONSTANTS OF SOIL BY A LECHER-WIRE METHOD AT A WAVE-LENGTH OF 1·5 M.

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ABSTRACT. The paper describes an investigation carried out by means of a laboratory method of measuring the electrical properties of soil with alternating current of a frequency corresponding to a wave-length of 1·5 m. A parallel Lecher-wire system was set up and coupled to a source of oscillations of the desired frequency. The length of the stationary waves set up on the wires in air was measured and compared with the corresponding wave-length when the wires were immersed in the sample of soil under examination. The ratio of these wave-lengths gives directly a quantity involving both the conductivity and the dielectric constant of the soil. This quantity was measured for typical samples of Teddington soil having a wide range of moisture contents, and from the results deductions are made as to the values of the conductivity and dielectric constant of soil for various moisture-content conditions. Thus for conditions of normal moisture content the dielectric constant is found to have a value of 10 or 12, while the conductivity lies within the range 10 to 28 $\times$ 10^8 e.s.u. These values are in good agreement with those obtained by the use of a field method on the same site and at the same wave-length, and their accuracy is adequate for most purposes in connexion with radio communication.

§ 1. OBJECT OF THE PRESENT INVESTIGATION.

In two previous papers[1], the authors have described an investigation of the mode of propagation of ultra-short electric waves along the earth's surface. Over the wave-length range of 1·5 to 10 m. employed in this work, the electrical properties of the earth play an important part in determining the rate of decrease of field-strength with increase in distance from the transmitter. A comparison was made between the experimental results obtained and theoretical attenuation curves calculated on the basis of Fresnel's equations governing the conditions of reflection of electric waves from the boundary of a medium such as the earth; and in this way the investigation led to a determination of the electrical properties (conductivity and dielectric constant) of the earth for the wave-length range in question. The accuracy of determination of these quantities has been considerably improved by an extension of these field experiments in the manner described in the previous paper by J. S. McPetrie[2]. Measurements were made by him on the electrical

constants of the ground under conditions of normal incidence and for wave-lengths of 1·5 and 0·46 m.

The object of the experiments described in the present paper was to determine the electrical properties of samples of soil by a laboratory method on one of the wave-lengths previously employed in the field work referred to above. At wave-lengths above 30 m. the use of a laboratory method[3] of measuring the conductivity and dielectric constant of samples of soil, or sea-water, has proved very successful as an alternative to the much more tedious and less direct field methods available for use on these longer wave-lengths. The method in question, however, which involved the measurement of the capacitance and resistance of a condenser filled with the sample under examination, is subject to considerable experimental difficulties when the wave-length is reduced to a few metres. An attempt was made, therefore, to study the properties of soil by observing the propagation of electric waves along a pair of wires embedded in a sample of the soil under investigation.

§ 2. THE PRINCIPLE OF THE METHOD EMPLOYED

The production of stationary electric waves on a parallel-wire system has been employed for the measurement of wave-length since the earliest days of radio-frequency research, for it was used in the classical work of Lodge and Hertz in 1888–1890 and later, and in more detail, by Lecher and Blondlot. This experimental method has also been applied by Heerwagen, Drude and others to the study of the dielectric constants of organic and inorganic liquids for wave-lengths of the order of 1 m.[4] In the majority of these cases, damped oscillations were employed originating in a spark discharge, although it is highly desirable that the oscillations should be continuous and have, as far as possible, a pure sine wave-form. Among the more recent workers Southworth[5] and Weichmann[6] have successfully used the ordinary Lecher-wire method with continuous oscillations, corresponding to wave-lengths between 0·1 and 3 m., to measure the dielectric constant of water. In later work, Drake, Pierce and Dow[7] have effected a refinement by using a concentric cylindrical system in place of the two parallel wires, and they have measured the dielectric constant of water with this arrangement for wave-lengths between 4 and 25 m.

In all this work the conductivity of the medium between the parallel wires was negligibly small, a condition which is no longer valid when the method is applied to the study of the properties of a material like soil. It is therefore useful to state briefly the theoretical basis of the method of employing stationary waves on parallel wires for the determination of the electrical properties of a medium which has an appreciable conductivity in addition to its dielectric constant.

It can be shown from the standard analysis of the propagation of alternating electric currents along a parallel wire transmission line that the voltage V_x at a distance x from the input end is given by an equation of the form

V_x, x

$$V_x = V_0 e^{-(\alpha + j\beta)x},$$

V_0, α, β

where V_0 is the input voltage to the line. In this equation α and β are known as

the attenuation and wave-length constants respectively. They are given by the equations:

$$\alpha^2 = \tfrac{1}{2}\left[\sqrt{\{(R^2 + \omega^2L^2)\,(G^2 + \omega^2C^2)\}} + (RG - \omega^2LC)\right] \quad \text{.....(1)},$$

$$\beta^2 = \tfrac{1}{2}\left[\sqrt{\{(R^2 + \omega^2L^2)\,(G^2 + \omega^2C^2)\}} - (RG - \omega^2LC)\right] \quad \text{.....(2)},$$

in which R, L, G and C are the primary constants of the line, i.e. the resistance, inductance, leakance and capacitance per unit length[8], and ω is $2\pi \times$ the frequency f. R, L, G, C ω f

When the line is immersed in a pure dielectric such as air or water, $R = G = 0$ and we have:

$$\alpha = 0 \quad \text{.....(3)},$$

$$\beta = \omega\sqrt{(LC)} \quad \text{.....(4)}.$$

Thus the length λ of the standing waves set up on the wire system is given by λ

$$\lambda = \frac{2\pi}{\beta} = \frac{1}{f\sqrt{(LC)}} \quad \text{.....(5)}.$$

Assuming the medium to have unit magnetic permeability, the inductance of the wires will always be the value for immersion in air, whereas the capacitance will be proportional to the dielectric constant of the medium. If with oscillations of the same frequency f the lengths of the standing waves set up on the wires are λ_a in air and λ_s when the wires are immersed in the medium under investigation, then λ_a λ_s

$$\frac{\lambda_a}{\lambda_s} = \frac{\sqrt{(L\kappa C)}}{\sqrt{(LC)}},$$

whence

$$\left(\frac{\lambda_a}{\lambda_s}\right)^2 = \kappa \quad \text{.....(6)},$$

where κ is the dielectric constant of the medium. κ

Now suppose the wires to be placed in a medium of conductivity σ, the leakance G will no longer be zero and equations (1) and (2) reduce to: σ

$$\alpha^2 = \tfrac{1}{2}\left[\omega L\sqrt{(G^2 + \omega^2C^2)} - \omega^2LC\right] \quad \text{.....(7)},$$

$$\beta^2 = \tfrac{1}{2}\left[\omega L\sqrt{(G^2 + \omega^2C^2)} + \omega^2LC\right] \quad \text{.....(8)},$$

in which, using the suffixes s and a to apply to the constants of the wires in the medium and air respectively, we have s, a

$$G_s = 4\pi C_a\sigma,$$

$$C_s = \kappa C_a.$$

Equations (7) and (8) now reduce to:

$$\alpha = \sqrt{\{\tfrac{1}{2}\omega^2L_aC_a\,(\kappa' - \kappa)\}},$$

$$\beta = \sqrt{\{\tfrac{1}{2}\omega^2L_aC_a\,(\kappa' + \kappa)\}},$$

where

$$\kappa' = \sqrt{(\kappa^2 + 4\sigma^2/f^2)}; \qquad \kappa'$$

and from equation (5)

$$\alpha = \frac{2\pi}{\lambda_a}\sqrt{\left(\frac{\kappa' - \kappa}{2}\right)} \quad \text{.....(9)},$$

$$\beta = \frac{2\pi}{\lambda_a}\sqrt{\left(\frac{\kappa' + \kappa}{2}\right)} \quad \text{.....(10)}.$$

These two equations specify the attenuation and wave-length constants respectively of the standing waves set up on the wire under the conditions specified.

From equation (9) it is seen that the attenuation constant of the line increases continuously as the conductivity of the medium between the wires increases from zero towards infinity. Thus, for highly conducting media such as concentrated electrolytes, the amplitude of the waves set up on the wires decreases rapidly with distance from the source.

Equation (10) shows that the wave-length when the wires pass through the medium is given by

$$\lambda_s = \frac{2\pi}{\beta} = \lambda_a \sqrt{\left(\frac{2}{\kappa' - \kappa}\right)},$$

whence

$$\left(\frac{\lambda_a}{\lambda_s}\right)^2 = \frac{\kappa' + \kappa}{2} = y, \text{ say} \qquad \text{.....(11).}$$

From this equation it is evident that a determination of the wave-length in air and in a medium such as soil gives the value of the quantity y or $\sqrt{\frac{1}{2}(\kappa' - \kappa)}$, which is a simple function of the electrical properties of the soil. If the attenuation constant of the waves could also be determined, the exact values of the conductivity σ and dielectric constant κ could be obtained with the aid of equation (9). The measurement of the attenuation constant is, however, more difficult than that of the wave-length although experiments in this direction have been made previously by M. Abraham[9] and by M. J. O. Strutt[10] at longer wave-lengths. Furthermore, exact determination of the actual values of σ and κ is not of very great practical importance, and the present investigation is limited therefore to the measurement of the wave-lengths in air and soil, and thus to a determination of the quantity $\frac{1}{2}(\kappa' + \kappa)$. A simple graph enables limiting values to be assigned to the quantities σ and κ to an accuracy which is sufficient for any practical case likely to arise in radio communication.

§ 3. EXPERIMENTAL PROCEDURE

The arrangement of the apparatus as finally adopted is shown in figure 1. Two hard-drawn copper wires of no. 14 s.w.g. were stretched horizontally at a distance apart of 2 cm. At one end the wires were terminated in a loop to enable the system to be coupled to the valve oscillator which was the source of the radio-frequency alternating current. At the other end the wires were connected through a non-inductive resistance equal to the surge impedance of the line, the effect of waves reflected at this end being thereby reduced. A wooden box of approximate dimensions 66 × 16 × 13 cm. was mounted as shown at (*a*), figure 1, with the two parallel wires threaded through thin keramot end-plates. This box could be slid along the wires so that the sample of soil under test could be placed in any desired position. The cross-sectional dimensions of the box are of some importance since, although the medium between the wires is the major factor determining the constants of the wires, these constants are also partly dependent upon the surrounding medium outside the wires. The effect of the surroundings decreases as either the

conductivity or the dielectric constant of the medium between the wires increases. The size of box used was adequate since it was found that the last few cm. of soil placed at the top of the box made no detectable alteration in the standing waves when the box was filled with dry soil, which is the most unfavourable case.

Three types of oscillator can be used in this experiment: (i) the ordinary retroaction type of valve oscillator such as was employed in the previous field work on a wave-length of 1·6 m.; (ii) an oscillator producing electronic oscillations on the Barkhausen-Kurz principle; and (iii) a magnetron oscillator. The oscillator of type (i) was found to produce only very weak oscillations at a wave-length of 1·5 m., this being its lower limit of operation. The electronic oscillator (ii), using a small cylindrical-electrode transmitting valve of the 40 to 250-watt type, was found to be a very convenient and simple source for demonstration purposes, and

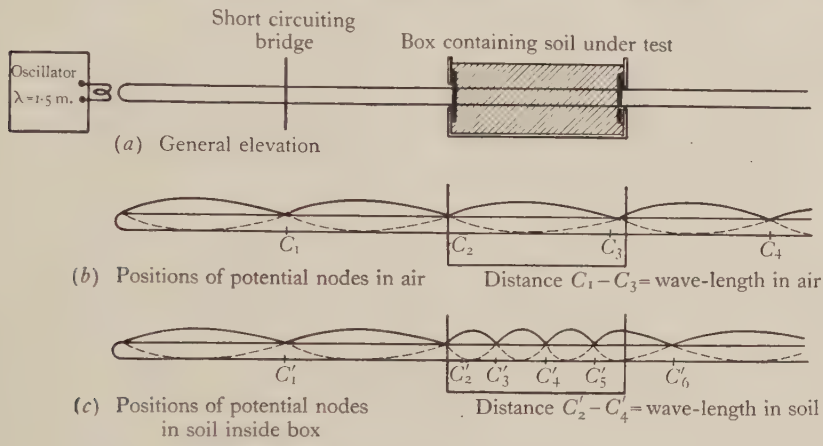

Figure 1. Diagrammatic illustration of standing waves on a Lecher-wire system in air and in soil.

was actually used at the Physical Society's Exhibition in January 1934[11], when the application of the Lecher-wire method to the determination of the dielectric constant of water was demonstrated. For more accurate working, however, it was desirable to have a greater amplitude of oscillation than is obtainable from this arrangement. A magnetron oscillator of the type described by E. C. S. Megaw[12] was employed therefore for the majority of the final measurements on soil. The input end of the Lecher-wire system was coupled to the inductance of this oscillator, and the presence of standing waves on the wires was indicated by a small change in the anode current supplied to the valve. To obtain increased sensitivity the steady current flowing was balanced out in the well-known manner, so that the relatively small changes could be observed with ease.

When the medium under examination is a liquid, as was the case in all the previous work referred to in § 2, it is a simple matter to locate the position of the potential nodes by sliding the bridge along the wires in the liquid. When the medium is soil, however, such a method would entail considerable disturbance of

the soil, and so the following procedure was adopted. First, with the box empty, the positions of the potential-nodes (or current-antinodes) were determined by short-circuiting the wires with a copper bridge. When the bridge was in position at a potential-node, such as C_1, C_2, ..., figure 1 (*b*), the Lecher-wire system was in resonance, and the increased load on the source was shown by the change in the deflection of the anode-current meter of the oscillator. The box was now moved so that its forward end coincided with the position of the node C_2; this ensured that there was no potential variation on the wires at this point, so that the effect of reflection at the boundary between soil and air would be reduced to a minimum. The distance between the successive nodes C_1, C_2, ... provided a determination of the wave-length in air employed for the experiment. The box was now filled with soil and a continuous watch was kept on the positions of the nodes C_4, C_5, ... on the wires on the side of the box remote from the oscillator. As the soil was placed around the wires the nodes C_4 and C_5 moved into the box into the positions C_4', C_5' as indicated at (*c*), figure 1. It was noted that this movement of the nodes was most affected by the soil placed in the immediate proximity of the wires, the soil at the bottom and near the top of the box having little effect. It was also observed that the nodes closed up appreciably as the soil was compressed, and the measurements were finally taken with the soil rammed as much as possible to simulate the conditions prevailing in the field.

When the box was filled with the sample of soil the positions of the nodes C_1' and C_2' were checked and the fresh position of C_6' was determined. If λ_a and
l, d λ_s are the wave-lengths in air and soil respectively, l is the length of the box and d the distance from the end of the box to C_6' at (*c*), figure 1, then we have:

n
$$\frac{l}{\lambda_s} = \frac{n\lambda_a - d}{\lambda_a},$$

where n is the number of whole wave-lengths between C_2' and C_6'.

Thus
$$\frac{\lambda_a}{\lambda_s} = \frac{n\lambda_a - d}{l} \qquad \text{.....(12),}$$

and hence from equation (11)

$$\kappa \quad \left(\frac{\lambda_a}{\lambda_s}\right)^2 = \frac{\kappa' + \kappa}{2} = \left(\frac{n\lambda_a - d}{l}\right)^2 \qquad \text{.....(13).}$$

The above procedure thus enables a precise determination of the wave-length in soil to be made without the potential nodes in the soil being actually located, a process which would obviously entail some disturbance of the packed soil.

§ 4. DISCUSSION OF RESULTS OBTAINED

After the apparatus had been assembled and put into working order, a check was made on the accuracy of the method by using water as the dielectric. From the results obtained by previous workers, to whom reference has already been made, it is considered that the dielectric constant of distilled water is known to be 79. at 20° C. for wave-lengths between 1 and 10 m. This preliminary test with water

showed that the experimental determination of the position of the nodes could be made to an accuracy of a few per cent, which is quite adequate for the soil measurements undertaken. After the preliminary work, the wave-length employed was 1·5 m. for all measurements, and it was found that the corresponding wave-length in a specimen sample of surface soil taken straight from the ground was about 0·35 m. This wave-length is very dependent upon the actual moisture content of the soil, since it was already known from the work at longer wave-lengths that this factor influences to a considerable extent the conductivity and dielectric constant of the soil.

Accordingly, repeated measurements were made in the manner described on two samples of soil the moisture of which was gradually removed by a slow drying process. These samples were taken from just below the surface of an open field at the National Physical Laboratory, Teddington, and while in one case the soil was used in the condition in which it was removed from the ground with a moisture content of about 18 per cent, the second sample was artificially moistened with water to bring its moisture content up to nearly 25 per cent. Simultaneously with the electrical measurements, which were made daily on the two samples, a determination by weight was made of the moisture content, the results of which are expressed as the proportion of water to dry soil. All the measurements were carried out at a room-temperature of about 15° C.

The results of the measurements made on these two typical samples of soil are given in the graph forming figure 2, which shows the square of the ratio of the wave-length in air to the wave-length in soil, i.e. the quantity y in equation (11), plotted against moisture content. It is seen that y decreases from a value of about 23 for very wet soil to about 4 for very dry soil.

For moisture contents between 12 and 16 per cent, the broken line in figure 2 indicates a departure from the normal course of the full-line curve. Over this range it appears that the variation of the wave-length in soil with moisture content is much less than that indicated by the main curve, and the nature of the curve suggests the possibility of some resonance effect in the medium. The detailed examination of this suggestion has not yet been possible and this point must be left for later investigation.

Now the quantity y involves the conductivity and dielectric constant, both of which may vary with frequency and moisture content. Thus it is difficult to separate the individual values of σ and κ in the expression for y or $\frac{1}{2}(\kappa' + \kappa)$, but some very useful deductions may be made for the conditions which are of most practical importance. In the first place, previous work at longer wave-lengths has indicated that as the soil is dried out its conductivity decreases so rapidly as to render its effect negligible for moisture contents below 3 or 4 per cent. Thus the value of y for very dry soil is virtually the minimum value of κ for dry soil. In a similar way the previous work has indicated that the value of κ increases considerably with moisture content, and presumably it tends towards the value of 80 for pure water as the proportion of soil decreases to zero. The value of the dielectric constant for soil having a normal moisture content of about 20 per cent

is therefore greater than 4, the value obtained for dry soil in the present experiments, and it is probably several times this value. Now in figure 3 curves have been plotted showing the relation between σf and κ for various specified values of the quantity y. Thus if we take y equal to 20 for normal soil from figure 2, we see from figure 3 that $\sigma/f < 18$ since we know that $\kappa > 4$. For a probable value 10 of κ the corresponding value of σf is 14. In this way we can determine a definite upper limit and a probable lower limit of σf. Since the wave-length of 1·5 m. employed corresponds to a frequency of 2×10^8 c./sec., the limiting values of the conductivity of the ground are 36×10^8 and 28×10^8 e.s.u.

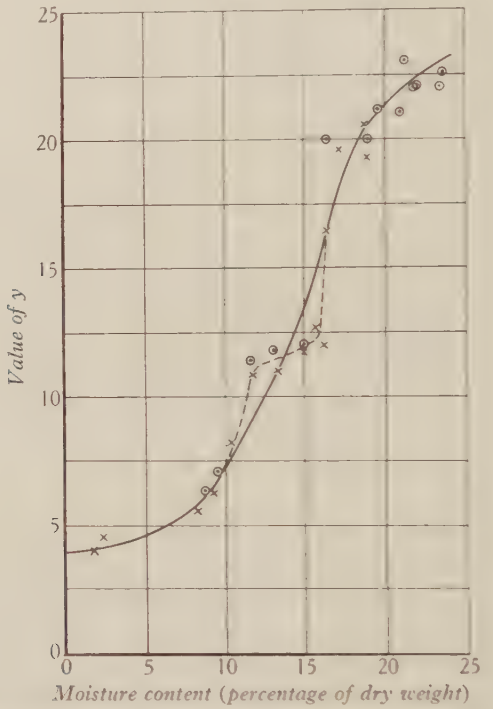

Figure 2. Relation between moisture content and value of y or $(\lambda_{air} \lambda_{soil})^2$. × Normal sample of surface soil. ⊙ Sample of soil artificially moistened.

In the preceding paper by J. S. McPetrie[2], describing the field measurements carried out on the same wave-length of 1·5 m., it was deduced that the dielectric constant for soil or grass was between 7 and 16 while the conductivity was about 10 10^8 e.s.u. These experiments were carried out in the summer when the moisture content may be taken as about 16 per cent, which is the lower limit found for Teddington subsoil over a typical period of one year. For this moisture content the value of y obtained from figure 2 is about 14; thus the dielectric constant cannot exceed 14 in this case, while for the mean value (between 7 and 16) of 12 the corresponding conductivity is about 10 10^8 e.s.u. It is thus seen that the results obtained by the field and laboratory methods of measuring the electrical properties of the soil at a wave-length of 1·5 m. agree reasonably well when the variation in

these quantities with moisture content is taken into consideration. In one of the authors' earlier papers the use of a less accurate field method led to a value of the conductivity as high as 95×10^8 e.s.u. This is a possible value which might be accounted for by high moisture content, but the latest determinations are considered to be more accurate. In any case, however, all three determinations indicate quite definitely that at a wave-length of 1·5 m. the conductivity of the surface soil at Teddington is in excess of 10×10^8 e.s.u. as distinct from the values less than 2×10^8 e.s.u. obtained for wave-lengths above 30 m. The intervening gap between 1·5 and 30 m. has not yet been satisfactorily filled by a laboratory method, but research in this region is now in progress.

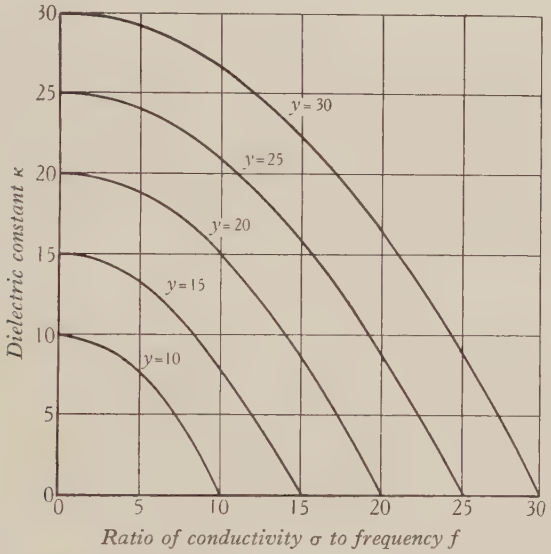

Figure 3. Relation between κ and σ/f for various values of y or $(\lambda_{air}/\lambda_{soil})^2$.

§ 5. ACKNOWLEDGMENTS

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THE THERMAL AND ELECTRICAL CONDUCTIVITIES OF METALS AND ALLOYS: PART 1, IRON FROM 0° TO 800° C.

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ABSTRACT. A longitudinal-flow method has been used to determine the thermal conductivity, at mean temperatures ranging from 30° to 800° C., of a nickel-plated rod of Armco iron containing approximately 99·92 per cent of iron. After allowance for the effect of the nickel plating, and extrapolation to 0° C., a value of 0·177 c.g.s. units is obtained for the thermal conductivity of the iron. This value is higher than that usually attributed to iron, but a chemically prepared iron of greater purity has been examined also and found to have a thermal conductivity of 0·194 c.g.s. units at 0° C., and it seems evident that the lower results obtained by earlier investigators were due to the relative impurity of their specimens. The conductivity of Armco iron decreases with increase in temperature, the value at 800° C. being 0·071 c.g.s. units. By the use of values of the electrical conductivity which were determined during the same experiment as the thermal conductivity, the Lorenz function of this sample of iron has been calculated. It is found to increase from a value of $0{\cdot}62 \times 10^{-8}$ at 0° C. to the unusually high value of $0{\cdot}74 \times 10^{-8}$ at 400° C. and to remain approximately constant over the temperature-range of 400° to 700° C.

§ 1. INTRODUCTION

Some eighty years have elapsed since Wiedemann and Franz* first propounded the law that the thermal conductivity of a metal is directly proportional to its electrical conductivity. In 1872 Lorenz† stated that the thermal conductivity was proportional to the product of electrical conductivity and the absolute temperature. This may be written

$$K/\sigma T = L, \qquad L$$

where K is the thermal conductivity in c.g.s. units; K
σ the electrical conductivity in $\text{ohm}^{-1}\text{-cm}^{-1}$; σ
T the temperature in degrees absolute. T

L has been called the Lorenz constant, and several experiments‡ have been made to test the relationship. It is approximately true for a considerable number of metals at atmospheric temperature, although some of the poorer conductors appear to give an abnormally high value for L. At low temperatures, however, most metals fail to conform to the law. It is preferred on this account to refer

* *Ann. der Phys.* **89**, 497 (1853).
† *Ann. der Phys.* **147**, 429 (1872).
‡ *Handbuch der Experimentalphysik*, **9**, pt 1, 207–11.

to L as the Lorenz function. Had the Wiedemann-Franz-Lorenz law been universally true, the relatively simple measurement of electrical conductivity or resistivity would have provided data for the calculation of the thermal conductivity and what is often a lengthy and difficult measurement could have been avoided. At present it is necessary to make accurate determinations of both the thermal and the electrical conductivity of as many conductors as possible over a wide range of temperature, in order to appreciate the range of validity of the above law and, perchance, to suggest modifications therein.

Two contributions to this subject have already been made from this laboratory. F. H. Schofield* made measurements on copper, magnesium, zinc, aluminium and nickel at temperatures ranging from about 100° C. to a maximum of 730° C. in the case of the nickel, and found that for the first three metals the Lorenz function was nearly constant at all temperatures while for the other two it increased with increase in temperature, a constant value being reached at about 300° C. in the case of nickel. E. Griffiths and F. H. Schofield† investigated a number of bronzes and light alloys from 80° C. to about 300° C. and found, with but one or two exceptions, that the value of the Lorenz function of the alloy was of the same order as that of the main constituent element. The method of experiment then employed to determine the thermal conductivity involved the measurement of the heat flowing through a rod of the material under test by means of a water-flow calorimeter attached to one end, and this definitely limited the range of temperature over which the determination could be made.

It was thought that the range of future experiments could be considerably increased by replacing the water-flow calorimeter by a metallic rod of known thermal conductivity. The following is an account of the determination over the range 0° to 800° C. of the thermal and electrical conductivity of a rod of Armco iron which is intended for use as such a thermal conductivity standard.

Choice of a thermal-conductivity standard. The chief requirements for a metal to be used as a thermal-conductivity standard over a wide range of temperatures are that it shall possess a high melting-point, remain unaffected by repeated exposure to high temperatures, and possess a thermal conductivity of the same order as that of the metals to be tested, which should either be independent of the temperature or vary approximately linearly with it. It is hoped to be able to carry out tests on some of the more poorly conducting metals which might be expected to have abnormally high values of the Lorenz function at room temperatures, and it appeared advisable for the standard material to possess a thermal conductivity between 0·03 and 0·3 c.g.s. units. The only pure metals which fall within this range and have a sufficiently high melting-point are iron, nickel and cobalt. Each of these metals possesses a magnetic transformation point at which a sudden change in conductivity might be anticipated. Cobalt has transformation points at about 460° C. and 1100° C. whilst that of nickel occurs at about 370° C., so neither of these metals can be used. The effect of the transformation point of iron should not

* *Proc. R.S.* A, **107**, 205–27 (1925).
† *J. Inst. Metals*, **39** (i), 337–74 (1928).

be unduly troublesome, for the point occurs at the upper end of the proposed temperature-range, and as iron of fairly high purity could readily be obtained in the form of Armco ingot iron, it was decided to investigate the thermal and electrical conductivity of this material.

§ 2. METHOD OF EXPERIMENT

The most suitable method for the determination of the absolute thermal conductivity of a metallic conductor at high temperatures appears to be one of the type employed by Schofield in the investigation to which reference has already been made. Heat is generated at a constant rate at the midpoint of a rod of the material under test. Leakage of heat to or from the surface of the rod is prevented by a surrounding guard tube, so that the whole of the heat supplied is operative in establishing a gradient of temperature towards each end. The measurement of these two gradients, the cross-sectional area of the rods, and the rate of supply of heat provide sufficient data for the calculation of the thermal conductivity. The author decided to use a method similar to this for the measurement of thermal conductivity and to arrange for measurements of the electrical conductivity to be made at the same time. Guided by the results of the above-mentioned work, the author considered it advisable to reduce the relative effect of any lateral gain or loss of heat, and thus to be in a position to measure the thermal conductivity of poorer conductors with greater accuracy, by using specimens about 3 in. in diameter, surrounded by a 6-in. guard tube. This increase in diameter naturally increased the heat capacity of the apparatus and thereby lengthened the time required for the attainment of steady conditions, but for the same gradient and conductivity it reduced the corrections per cm. for lateral heat-interchange to about one-tenth of those obtaining in the aforementioned experiments. A second advantage was that the temperature-gradient could be determined with greater certainty because, as will be explained later, it became practicable to measure all of the required temperatures by means of the same interchangeable thermocouple.

§ 3. PREPARATION OF SPECIMEN

Two similar rods of Armco ingot iron, each 3 in. in diameter and 42 in. long, were procured from the Armco International Corporation. An analysis of the turnings taken over a complete cross-section of the rod gave the following percentage chemical composition*.

Carbon	0·023
Silicon	0·007
Sulphur	0·020
Phosphorus	0·007
Manganese	0·025
Iron (by difference) ...	99·918

* The author is indebted to Mr T. E. Rooney of the Metallurgy Department for this analysis.

One end of each rod was recessed to receive a heating-coil, as shown in figure 1(*a*), and was turned down to a mean diameter of 2·84 in. over a length of 15 in. At distances of approximately 6 and 25·4 cm. from this end holes 0·2 cm. in diameter were drilled to the centre of the rod to receive the thermocouple. The rods were then heavily nickel-plated over this machined portion by Messrs Fescol Ltd., and then again machined so as to have a final diameter of 2·895 in.

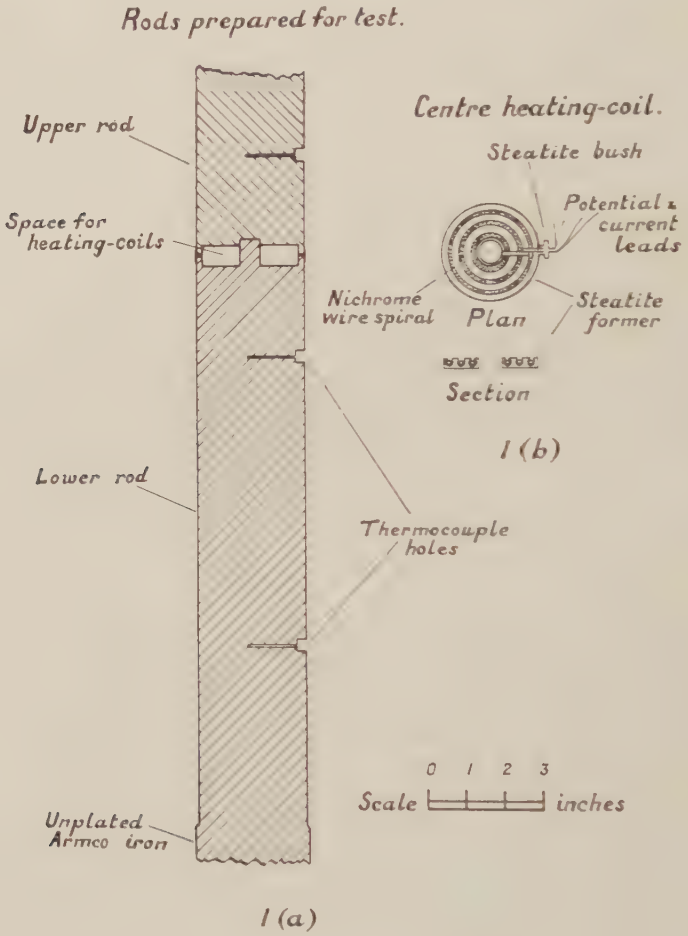

Figure 1.

Two similar heating-coils, each of approximately 22 ohms resistance, were prepared. One of these coils is shown in figure 1 (*b*). It consisted of a spiral of nichrome wire held in three concentric circular grooves cut in a disc of steatite which fitted closely into the cavity in the end of the rod. The steatite walls of these grooves converged at the top and projected above the nichrome wire, so as to prevent the possibility of short circuit between the wires and the specimen. The leads to each coil passed out through a specially designed insulator of baked steatite

inserted in a hole drilled in the side of the recessed portion in the rod. At a distance of approximately 1 cm. beyond this steatite insulator, current and potential leads were welded to the wires from the heating-coil, an atomic hydrogen arc being used for this purpose. The current leads consisted of 10 strands of nichrome wire of the same gauge twisted together, and the potential leads of a single strand of similar wire. When the two iron rods were mounted end-to-end the two heating coils became enclosed at the centre of the composite rod. Lateral displacement was prevented by means of a centre pin in the form of a $\frac{1}{2}$-in. rod of iron. Holes $\frac{1}{2}$ in. in diameter were drilled through each rod at distances of about 2 in. from each unmachined end. These were to facilitate assembly and to provide sockets in which to insert the plugs of the current leads required for making the electrical conductivity measurements.

§ 4. DESCRIPTION OF FURNACE

The furnace consisted of a central iron tube $\frac{1}{4}$ in. thick, 6 ft. long and of 6 in. internal diameter, threaded for about a foot at the lower end to receive a heavy flange. The latter was bolted to a 2-ft. square base plate of iron $\frac{1}{2}$ in. thick which was supported at the corners by stout levelling-screws resting on four oak blocks 15 in. in height and bolted together by rods of $\frac{1}{2}$-in. iron so as to form a rigid stand. A hole 7 in. in diameter had been cut in the centre of the base plate, and the tube was screwed down until about 9 in. projected below the base plate. This provided an exposed surface from which the rate of loss of heat could be readily controlled. A similar length was left exposed at the top of the tube, and the remaining central portion was covered with micanite and uniformly wound with nichrome tape. The windings of the furnace were tapped so as to be in seven sections which could be independently controlled, so that the temperature-distribution of the specimen could be accurately matched by that of the furnace tube. As a further aid towards this end an additional narrow heating-coil was wound on the centre section of the furnace tube. The turns of wire were so spaced as to enable four $\frac{3}{8}$-in. holes to be drilled in a vertical plane of the furnace wall and in positions to coincide with the thermocouple holes drilled in the specimen. The whole of the windings were then covered with a layer of alundum cement.

Cylinders of sheet iron 11 in. and 23 in. in diameter, provided with holes for the passage of the heating-coil leads and the interchangeable specimen thermocouples, were then placed in position around the furnace tube. The nichrome leads were insulated by means of steatite bushes at the points where they passed through these cylinders and were finally bolted to a terminal board fixed in front of the furnace. A silica tube to serve as a sheath for the interchangeable thermocouple was inserted from the outside into each of the four holes in the furnace tube. The space between the furnace tube and the inner cylinder was then packed with sil-o-cel powder, and that between the inner and outer cylinders was packed with magnesia asbestos lagging.

Several thermocouples composed of 0·02-cm. platinum and 9 : 1 platinum-rhodium wires had previously been pegged into the inner wall of the furnace. These were insulated with twin bore silica tubing and the free ends of the wires were brought out at the top of the furnace.

§ 5. ASSEMBLY OF APPARATUS

The two halves of the specimen having been put together to form one continuous rod, with the two independent heaters enclosed in the central plane, six thermocouples composed of wire similar to the above were pegged into the test portion of the specimen and were insulated with twin-bore silica tubing supported alongside the rod so that the free ends were brought to the top. The method of pegging these couples consisted in welding the wires in an oxyhydrogen flame until the blob was just large enough to be firmly held in a small hole drilled in the surface of the rod by means of a no. 76 drill. The leads from the heating-coils were insulated with silica tubing and brought to the top. The specimen was then supported by means of a detachable framework and lowered into the furnace to rest on a small levelling table. The framework was removed, and the specimen was adjusted until it stood centrally within the furnace and the four tubes to protect the thermocouple could be inserted in their appropriate holes. The interspace between the specimen and furnace was finally packed with sil-o-cel powder to prevent the formation of convection currents. The apparatus so assembled is shown in figure 2. It will be observed that the ends of both specimen and furnace tube remain exposed to provide means for the control of the rate of loss of heat, additional cooling or heating units being fitted to these ends as required.

The circuit for the determination of the electrical conductivity was completed by connecting leads from the 110-volt d.-c. mains to the top and bottom of the specimen and including in the circuit a standardized resistance of 61·9 $10^{6}\Omega$. and a controlling rheostat.

§ 6. DESCRIPTION OF THE EXPERIMENTS

To determine the thermal conductivity it was necessary to supply heat to the centre of the specimen and furnace in order to impose a gradient of temperature towards each end, and to adjust the various currents in such a way that when the steady state was attained corresponding points of specimen and furnace were at the same temperatures. This adjustment minimized any lateral transfer of heat and the whole of the heat generated at the centre of the specimen was then assumed to be operative in establishing the observed temperature-gradient. The temperature of the specimen was obtained by means of a thermocouple insulated by a 15-in. length of very fine twin-bore silica tubing which could be introduced through each of the four silica sheaths into the centre of the rod in these four positions. The electrical energy generated in the centre specimen-heaters was obtained by measuring the potential-drop across each coil and determining the current flowing

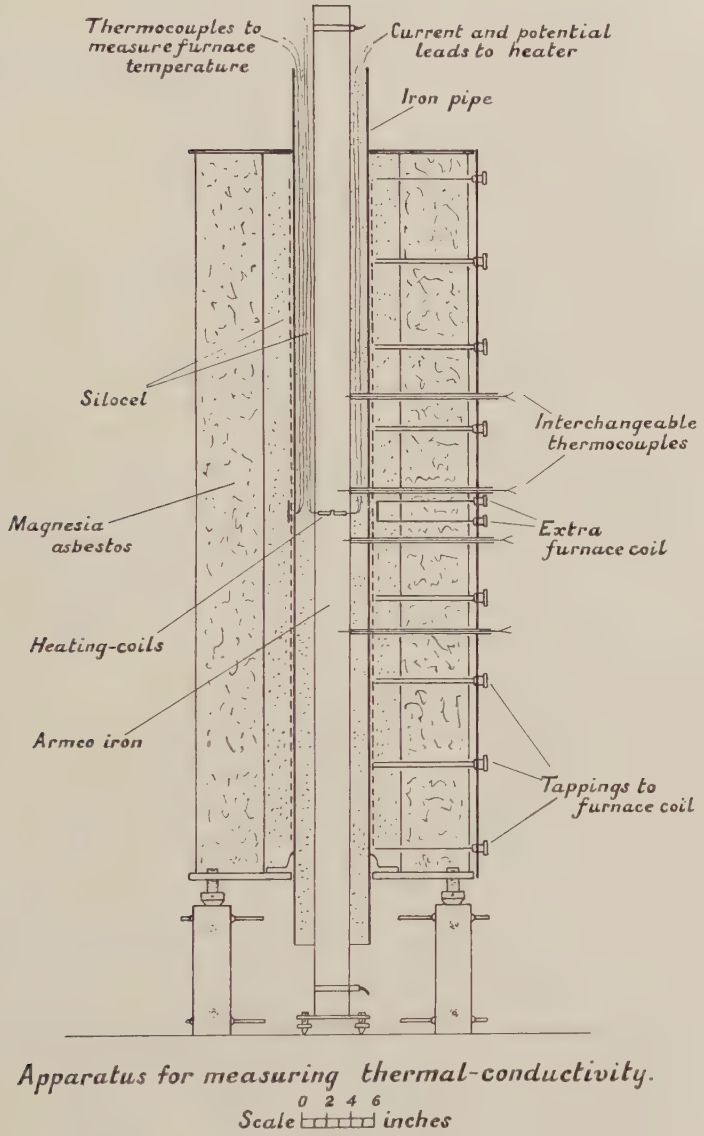

Figure 2.

in each in terms of the potential-drop across a standard shunt included in the same circuit. The thermal conductivity was calculated from the formula

$$K_{TS} = \frac{E_1 C_1 + E_2 C_2}{JA\,\{(t_2 - t_1) + (t_3 - t_4)\}/l} \text{ c.g.s. units,}$$

K_{TS}, T_S where K_{TS} is the thermal conductivity at the mean temperature T_S;

E_1, E_2 E_1 and E_2 are the potential-drops across the two coils (V.);

C_1, C_2 C_1 and C_2 are the currents in the two coils (A.);

A A is the area of the cross-section of the rod (cm^2);

t_1, t_2, t_3, t_4 t_1, t_2 and t_3, t_4 are the two temperature-measurements on the upper and lower halves (° C.);

l l is the distance between these pairs of couples (cm.); and

J J is the Joule equivalent (assumed to be 4·184).

It was not always possible to obtain an exact agreement between the temperature of the furnace and specimen, so at each mean temperature apparent values were obtained for the thermal conductivity when the mean temperature of the furnace was a few degrees on either side of that of the specimen. These were then plotted to give the true thermal conductivity for an exact temperature-match.

Simultaneous measurements were made of the electrical resistance, a current of the order of 60 A. being employed for this purpose; and measurements of the potential drop on the specimen were obtained by inserting iron wires into the four thermocouple holes, readings being taken before and after reversal of the current so that the thermal e.m.f. due to the temperature-gradient in the rods could be eliminated. Detailed results are tabulated in the appendix, and all experimental values of the true thermal conductivity are shown plotted against the mean temperature in figure 3. The values of the electrical conductivity are similarly plotted in figure 4.

The first two runs were restricted to temperatures below 650 C. on account of failure of the centre heating-coils. In the third run this trouble occurred again after the specimen had been maintained at a temperature of about 800 C. for some days, and as steady matched conditions had not then been satisfactorily attained it was only possible to obtain electrical-conductivity data at the highest temperatures. The fourth run was without mishap and in addition to thermal and electrical conductivity-measurements up to 800 C., check values during the final cooling were obtained. Between each run the apparatus was dismantled and reassembled. Some of the thermocouples which had been pegged into the furnace wall were broken during this process, and were replaced by new thermocouples either hanging freely close to the furnace wall, or moveable within lengths of thin-walled silica tubing. Reasonably good agreement was obtained between all three types of furnace couples. An examination of the experimental points plotted in figures 3 and 4 fails to reveal any serious change in either the thermal or the electrical conductivity occurring as a result of the heat treatment involved in the foregoing tests, from which it is concluded that the conductivity of the rods should remain unaltered in subsequent comparative tests within the range 0° to 800° C.

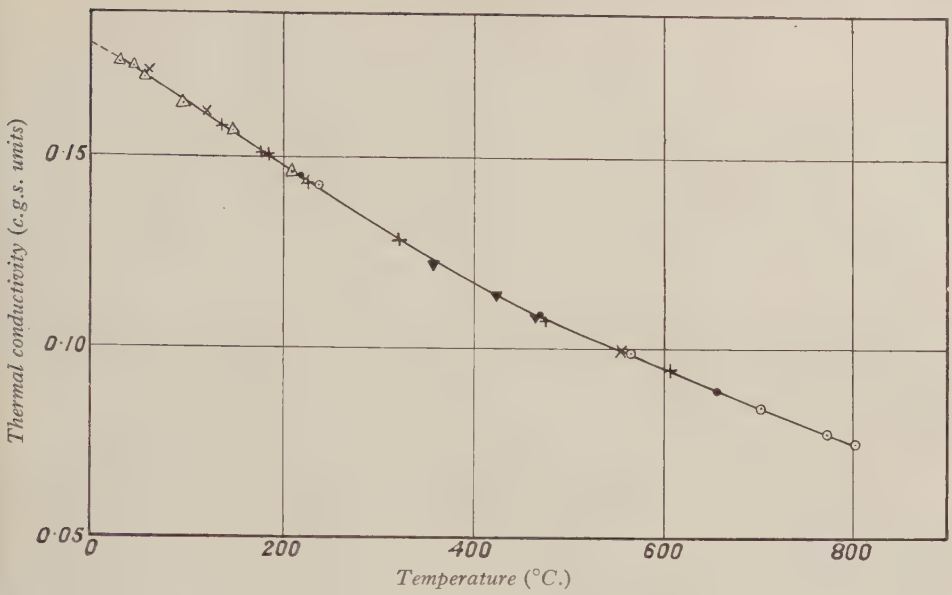

Figure 3. Variation of thermal conductivity of Armco iron (nickel-plated) with temperature.

First run: heating + ; second run: cooling ▼ ; third run: heating × ;
fourth run: heating ⊙, cooling • ; alternative method △.

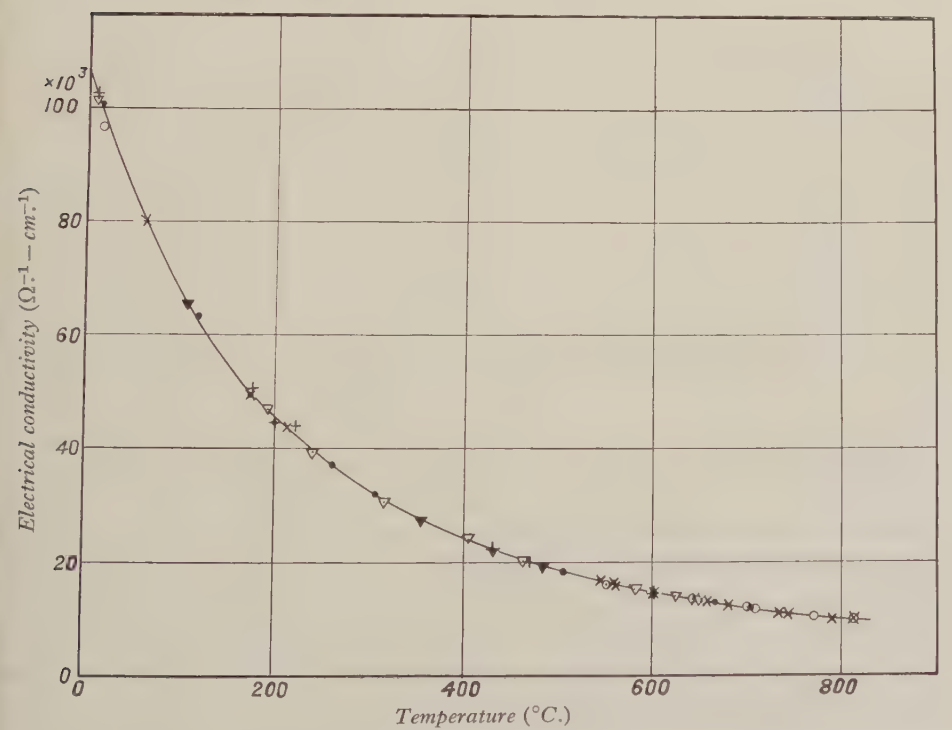

Figure 4. Variation of electrical conductivity of Armco iron (nickel-plated) with temperature.

First run: heating +, cooling +; second run: heating ▽, cooling ▼;
third run: heating ×, cooling ×; fourth run: heating ⊙, cooling •.

§ 7. ALTERNATIVE METHOD USED OVER THE RANGE 30° TO 200° C.

A few additional determinations were made on one of the rods in which the energy was measured calorimetrically. The water was supplied at a constant rate and temperature to a cooling unit soldered to the plated end of the specimen. The increase in the temperature of the water during its passage through the cooling unit was measured by means of a differential thermocouple and it served, together with a knowledge of the rate of water flow, to determine the quantity of heat flowing from the rod. Precautions were taken to prevent lateral flow of heat by surrounding the rod with a guard tube. The values of the thermal conductivity obtained by this method also are plotted in figure 3. It will be seen that they are in good agreement with those obtained in terms of the electrical input, and that the method has enabled the experiments to be extended down to 30° C.

§ 8. ERRORS ARISING IN FOREGOING EXPERIMENTS

Every effort has been made to reduce to a minimum the lateral interchange of heat between the furnace and specimen. Apart from this there are the following possible sources of error to be considered: (i) loss of heat by conduction along the thermocouples and the current and potential leads; (ii) loss of heat by conduction in sil-o-cel insulation; (iii) loss of measured heat due to the fact that the potential leads were welded to the heating-coils at a point some 2 cm. outside the specimen; (iv) gain of heat due to the electrical energy generated in the leads to the heating-coils.

(i), (ii) and (iii) would tend to make the experimental value of the thermal conductivity too high, while (iv) would make it too low. The following paragraphs give an estimate of the probable error due to each cause.

(i) As all fixed thermocouples were brought out at the top of the apparatus, the loss of heat by conduction along those pegged into the upper half of the rod would be neutralized by the gain of heat by conduction along those pegged into the lower rod. All current and potential leads were also brought out at the top, and the gradient along them may be assumed to be the same as that of the top half of the specimen. Then, if the conductivity of nichrome is assumed to be comparable with that of iron, the conduction along the leads and the specimen will be proportional to their respective areas. The total number of strands of wire in the leads was 44, and as the diameter of the wire was 0·04 cm. the ratio of the areas is 0·0013. This indicates a probable error of 0·06$_5$ per cent since only half of the heat supplied has been considered. It is, of course, possible that the temperature-gradient in the current leads near the weld may be a little higher than that of the specimen at that point, but on the other hand the thermal conductivities of the leads and iron have been assumed to be equal, whereas at room temperature the conductivity of nichrome is about 16 per cent of that of iron, and at 800° C. it is some 75 per cent of the conductivity of iron. The heat conducted along the silica tubing insulating the current and potential leads is also quite small, since the area of the silica was only 0·3 cm^2 and its conductivity according to Seeman* varies from 0·0030 c.g.s.

* *Phys. Rev.* **31**, 119–29 (1928).

units at 100° C. to 0·0057 at 800° C. It does not therefore seem probable that the error due to this cause can amount to more than 0·1 per cent.

(ii) The area of the sil-o-cel insulation is about three times that of the specimen. Its thermal conductivity probably varies from 0·00015 c.g.s. units at 100° C. to 0·00030 at 800° C. So, if one-third of the heat flowing in the insulation is assumed to have emanated from the specimen and the other two-thirds from the furnace, the error would amount to 0·1 per cent at room temperature and 0·4 per cent at 800° C.

(iii) The four potential leads were welded at distances away from the specimen ranging between 1·5 and 2·0 cm., so that the length of heating-coil outside the specimen would not exceed 8 cm. The total length of wire in the two heaters was 8 m., so that 1 per cent of the measured energy is actually generated just outside the specimen. As some of this heat would probably enter the specimen, the error on this account may be taken to be less than 1 per cent.

(iv) The current leads consisted of 10 strands, twisted together, of wire similar to that constituting the heating-coil and were found to have a resistance per unit length of approximately one-eighth that of the single wire. There were four such leads passing along the length of the top half of the specimen and carrying the same current as the heating-coil itself; that is, 100 cm. of current leads were alongside the working section of the upper half of the specimen. Therefore 1·6 per cent of the measured energy is generated in a wire alongside the specimen. A considerable proportion of this extra unmeasured heat might be expected to enter the specimen, but as this energy is distributed uniformly along the length, and the mean temperature-gradient has been measured, the error likely to be introduced would be less than $\frac{16}{25}$ of the above, i.e. less than 1 per cent. It would seem, therefore, that the total errors due to these four causes practically cancel one another.

There are also the following possible sources of error which do not involve heat leakage: (v) non-rectilinear flow of heat; (vi) incomplete accuracy of temperature-measurements; (vii) incomplete accuracy of electrical measurements; (viii) incomplete accuracy of linear measurements; (ix) change of dimensions with temperature; (x) effect of temperature-gradient.

(v) The heating-coils embedded in the centre of the specimen were designed to supply heat approximately uniformly over the cross-sectional area of the rods, and a distance of about 6 cm. was allowed before observation of the upper temperature, to enable rectilinear flow of heat to become established. Experimental confirmation that the flow of heat was rectilinear was given by an exploration of the temperature-distribution within each of the four thermocouple holes, uniformity of temperature being obtained in each instance.

(vi) The thermocouples were initially calibrated at the melting-points of mercury, sodium chloride and gold, and the boiling-points of water, naphthalene, benzophenone and sulphur. A small amount of contamination occurred in the

course of the experiments and this was probably due to unavoidable contact with the iron, but it was prevented as much as possible by efficient insulation with unbroken silica tubing. The calibration of the interchangeable thermocouple was frequently checked, and its junction was renewed from time to time. Measurements of the furnace-temperature were also made with this thermocouple, so that a true correspondence was obtained between the temperatures of the furnace and specimen.

(vii) The electrical measurements were correct to at least 1 part in 1000.

(viii) The holes into which the interchangeable thermocouple was inserted were 0·2 cm. in diameter and 19·4 cm. apart, and therefore error due to variation in the actual position of the thermojunction could not exceed 1 per cent. In practice multiple readings which would reduce this error were taken. No permanent change occurred in the dimensions of the working sections of the rods as a result of the heat treatment, but it may be mentioned that the unplated furnace tube became badly oxidized, and as a result its length increased by about 1 per cent.

(ix) If the value given in the *International Critical Tables* for the coefficient of linear expansion of iron is applied, the true thermal conductivity at the maximum temperature of 800° C. would be about 1 per cent lower than the value calculated from the dimensions at atmospheric temperature. As this effect is within the limits of experimental error, and will be partially neutralized in the contemplated comparison tests by the expansion of the material under investigation, no correction has been made on account of it.

(x) Both the thermal and the electrical conductivities have been measured in the presence of a temperature-gradient, and the values obtained have been assumed to be those for the mean temperature of the specimen. This is strictly true only when the conductivity is independent of temperature or varies with it in a linear manner. In the present instance the thermal conductivity decreases approximately linearly with increase in temperature, but the electrical conductivity does not do so. As, however, the temperature-gradient seldom exceeded 50° C., it is not anticipated that any appreciable error is introduced on this account. After consideration of all the likely sources of error, it would appear that the experimental values of both the thermal and electrical conductivities of the nickel-plated specimen of Armco iron are correct to within 2 per cent over the temperature-range 0° to 800° C. The values of the thermal and electrical conductivities obtained at intervals of 100° C. from the smooth curves drawn through the points in figures 3 and 4 are tabulated in columns 2 and 3 respectively of table 1.

§ 9. DISCUSSION OF THE RESULTS

A number of earlier determinations have been made of the thermal conductivity of iron, and it would be of interest to compare these with the results of the present investigation. Before this can be done it is necessary to apply a correction to allow for the presence of the nickel plating. In order to do this the values obtained for

the thermal and electrical conductivities of a sample of electrolytic nickel submitted for test by Messrs Castner Kellner Ltd. have been employed*.

The value of the thermal conductivity K, electrical conductivity σ, and specific resistance ρ, of the nickel-plated specimen and of the sample of Armco iron after allowance has been made for the effect of the nickel are tabulated for intervals of 100° C. in table 1. K, σ ρ

Table 1

Temperature (° C.)	Nickel-plated Armco iron			Armco iron			
	K (c.g.s. units)	σ (ohm^{-1}-cm^{-1})	ρ (ohm-cm.)	K (c.g.s. units)	σ (ohm^{-1}-cm^{-1})	ρ (ohm-cm.)	L or $K/\sigma T$
0	0·179	106,500	$9{\cdot}4 \times 10^{-6}$	0·177	104,300	$9{\cdot}6 \times 10^{-6}$	$0{\cdot}62 \times 10^{-8}$
100	·164	67,400	14·8	·163	66,400	15·0	·66
200	·148	45,000	22·2	·147	44,300	22·6	·70
300	·132	32,300	31·0	·132	31,800	31·4	·72
400	·117	23,600	42·4	·116	23,200	43·1	·74
500	·105	18,500	54·0	·103$_5$	18,100	55·3	·74
600	·095	14,800	67·5	·093	14,300	69·8	·74
700	·085	12,000	83·3	·082	11,500	87·0	·73
800	·075	10,000	100·0	·071	9,480	105·5	·70

It will be observed that the correction which is necessary to allow for the effect of the nickel-plating becomes greater at the higher temperatures and amounts to about 5 per cent at 800° C. Below 400° C. the correction is small. The thermal conductivity of an unplated portion on one of the 3-in. rods was determined over the range 50° C. to 120° C. by means of the alternative method using the water-flow calorimeter. The results are included in the appendix, and it will be seen that the values obtained agree almost exactly with those for the nickel-plated portion. Several independent measurements of the electrical resistance of small rods of the Armco iron have since been made, and the value at 800° C. has been found to range from 104 to 106 $\times 10^{-6}$ ohm-cm. It is considered that the tabulated values for the thermal and electrical conductivity of Armco iron are correct to within 2 per cent.

The value of the thermal conductivity when extrapolated from 30° to 0° C. is 0·177 c.g.s. units, a value which is considerably higher than the commonly accepted value for pure iron. The weighted mean value of the thermal conductivity of iron according to the *International Critical Tables* is 0·148 c.g.s. units, whilst the range of probable values is considered to be 0·136 to 0·160 c.g.s. units. In this connection it may be remarked that insufficient attention appears to have been given to the state of purity of the iron. Among the earlier experimenters Lorenz† and Grüneisen‡ have found the thermal conductivity to be 0·165 and 0·172 respectively at 18° C.

* The author desires to acknowledge his indebtedness to Messrs The Imperial Chemical Industries (General Chemicals) Ltd., for permission to make use of these data, which it is hoped to publish in a later part of the work on this subject.

† *Wied. Ann.* **13**, 438 (1881).

‡ *Ann. der Phys.* **3**, 43 (1900).

and it is significant that the electrical resistances of their specimens at this temperature were 10·65 and 10·1 × 10^{-6} ohm-cm. These resistance values are lower than those of the majority of early investigators and suggest that the specimens employed were of relatively higher purity. In recent years this question of purity has been studied more closely. Benedicks, Bäckström and Sederholm* investigated a series of carbon steels; the purest, which contained 0·08 per cent of carbon, gave a resistivity of 10·55 × 10^{-6} at 18° C. and a thermal conductivity of 0·187 at 40° C. An analysis of the results indicated that pure iron should have a thermal conductivity of 0·227 at 40° C. and a specific resistance of 7·6 × 10^{-6} at 18° C. A similar investigation has been made by Masumoto† at a temperature of 34° C., the thermal conductivity of pure iron being given as 0·1741 and the specific resistance as 10·44 × 10^{-6}. Kannuluik‡ has recently investigated the thermal and electrical conductivity of Armco iron in the form of wire. The measurements were mostly carried out at low temperatures. At 0° C. his value for the electrical conductivity agrees closely with that of the present work, but his value for the thermal conductivity is about 5 per cent lower, being 0·1688 c.g.s. units. Sedström§ has obtained a value of 0·18 for a very pure electrolytic iron. Specimens of electrolytic iron have also been studied at low temperatures by Grüneisen and Goens‖ and by Eucken and Dittrich¶. The former obtained values of the specific resistance at 0° C. ranging from 8·71 to 9·95 × 10^{-6} but he records no value of the thermal conductivity at this temperature. Eucken and Dittrich were primarily interested in the effect of grain-size on the conductivities. They gave values at 0° C. of 9·6 × 10^{-6}, 10·3 × 10^{-6} and 10·65 × 10^{-6} for the specific resistance and 0·225, 0·214, 0·196 for the corresponding thermal conductivities. These latter values appear to be remarkably high for the electrical conductivities with which they are associated.

It would seem from the foregoing survey of recent experimental work that the thermal conductivity of pure iron at normal temperatures is considerably higher than it was believed to be a matter of ten years ago. In this connection it is of interest to record that the present author has recently determined the thermal and electrical conductivities of a sample of very pure iron which had been chemically prepared in the Metallurgy Department of the National Physical Laboratory, and has obtained values of 0·185 c.g.s. units and 11·45 × 10^{-6} ohm-cm. at 50° C. which upon extrapolation indicate values of 0·194 c.g.s. units and 8·8 × 10^{-6} ohm-cm. at 0° C.

With regard to the data at high temperatures, so far as the author is aware the present thermal-conductivity determination is the first which has been made with a relatively pure sample of iron**. Honda and Simidu†† determined the thermal and electrical conductivities of a number of carbon steels up to about 900° C. but the

* *J. Iron and Steel Inst.* **114** (ii), 127–75 (1926).
† *Sci. Rep. Töhoku Univ.* **16**, 417 (1927).
‡ *Proc. R. S.* **141**, 159 (1933).
§ *Inaug. Diss.* (*Lund*) *Stockholm* (1924).
‖ *Z. f. Physik*, **44**, 615–42 (1927).
¶ *Z. Phys. Chem.* **125**, 211–28 (1927).

** Since this paper was written results for the thermal-conductivity of several irons over the temperature range 100 to 500° C. have been published by S. M. Shelton, *B. S. J. of Res.* **12**, 441 (1934). The thermal conductivity of the purest sample studied, which contained 99·90 per cent of iron, is in close agreement with that of the present investigation.

†† *Sci. Rep. Töhoku Univ.* **6**, 219 (1917).

specific resistance of their purest iron (Swedish iron) at 0° C. was as large as $15{\cdot}3 \times 10^{-6}$, and the thermal conductivity of this sample decreased from 0·134 at 30° C. to 0·077 c.g.s. units at 800° C. It will be observed that the conductivities agree fairly closely at high temperatures, a fact which will be more fully dealt with in a later publication.

§ 10. THE LORENZ FUNCTION

The values obtained of the Lorenz function for Armco iron have been recorded in the last column of table 1 and are plotted in figure 5, in which the values of a number of other investigators also are included. The curve for copper, based on the results of Meissner* and Schofield†, are included for comparison. It will be

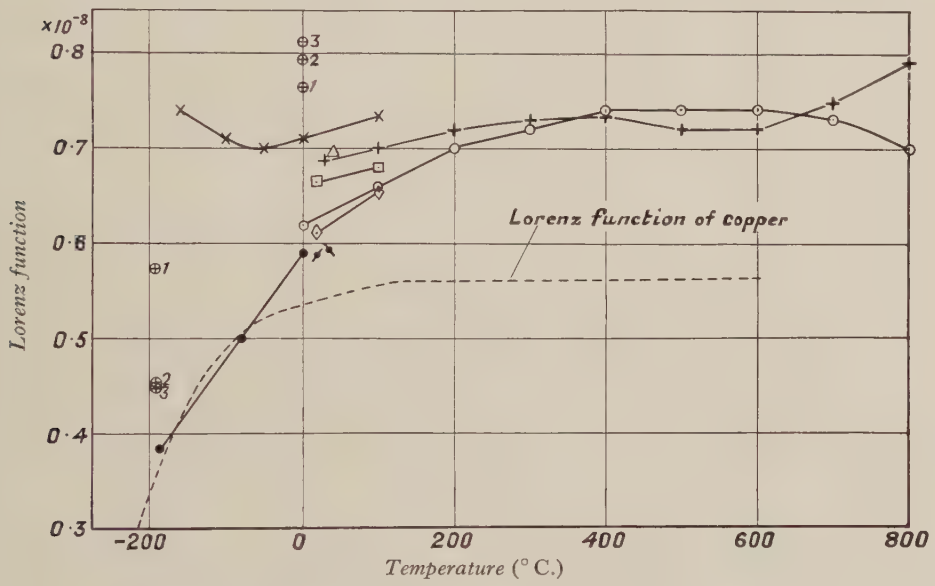

Figure 5. Variation of Lorenz function of iron with temperature.

⊕ Eucken and Dittrich. ✓ Grüneisen. + Honda and Simidu. ⊡ Jaeger and Diesselhorst. • Kannuluik. × Lees. ◇ Lorenz. ↘ Masumoto. △ Sederholm. ⊙ Present author.

observed that a considerable range of values has been obtained and that whereas the Lorenz function of the impure iron examined by Lees‡ increases as extremely low temperatures are approached the Armco iron tested by Kannuluik behaves similarly to copper at low temperatures in that the Lorenz function decreases rapidly with decrease in temperatures.

In the case of copper the value of the function becomes practically constant over the range from atmospheric temperature to 600° C. but the present experiments show the value for iron to be somewhat greater at normal temperatures and

* *Deutsch. Phys. Ges. Verh.* **16**, 262 (1914). † *Loc. cit.*

‡ *Phil. Trans.* A, **208**, 381 (1908).

to continue to increase with increasing temperature until it attains the unusually high value of $0{\cdot}74 \times 10^{-8}$ at about 400° C. It remains approximately constant at this value until between 600° and 700° C. when it commences to decrease slightly.

§ 11. ACKNOWLEDGMENTS

In conclusion the author desires to acknowledge his indebtedness to Dr G. W. C. Kaye, Superintendent of the Physics Department, for his encouraging interest in the work; to Dr F. H. Schofield for advice and assistance in the initial stages, and to Mr F. G. Filby and his successor Mr E. E. Smith, B.Sc., for assistance with the constructional and observational work.

APPENDIX

Experimental data for the thermal and electrical conductivity of a rod of nickel-plated Armco iron

Energy (W.)	Temperature (° C.) t_1	t_2	t_3	t_4	$(t_2-t_1)+(t_3-t_4)$	$\frac{t_1+t_2+t_3+t_4}{4}$ or T_S	T_F-T_S*	Apparent thermal conductivity K_{TS}	True thermal conductivity	Electrical conductivity σ_{TS}
First run										
						9·0				102,400
67·78	122·2	145·7	134·4	123·1	46·9	134·7	1·9	0·1578		
70·60	125·7	149·8	150·2	125·7	48·6	137·8	−1·2	0·1586		
67·37	124·3	147·8	148·2	125·0	46·7	136·3	0·7	0·1575		
						136·3			0·1582	
67·92	165·0	190·1	190·5	166·4	49·2	178·0	1·2	0·1507		
73·47	160·8	189·2	192·6	167·8	53·2	177·6	−1·1	0·1508		50,420
73·01	160·4	188·4	192·1	167·7	52·4	177·1	−0·4	0·1522		
						177·6			0·1512	
73·59	170·0	197·1	198·5	172·2	53·4	185·0	−0·2	0·1505		
73·69	171·6	198·4	199·0	172·7	53·1	185·4	−1·0	0·1515		
73·32	170·7	197·8	199·1	172·8	53·4	185·1	0·2	0·1500		
68·30	172·5	196·7	195·0	169·7	49·5	183·5	1·3	0·1507		
70·87	167·1	193·6	195·2	170·1	51·6	181·5	1·7	0·1500		
74·68	181·0	207·4	204·8	176·8	54·4	192·5	−3·3	0·1499		
						185·5			0·1505	
98·57	211·1	248·2	249·4	211·5	75·1	230·0	−2·6	0·1434		
93·87	209·5	245·4	246·5	210·4	72·0	228·0	−1·1	0·1424		
94·43	208·9	244·5	245·9	209·5	72·0	227·2	0·3	0·1433		
94·48	208·0	243·5	244·9	208·5	71·9	226·2	0·1	0·1434		
94·40	208·0	243·5	244·9	208·7	71·7	226·2	0·1	0·1437		
92·85	204·1	239·5	241·1	205·3	71·2	222·5	0·1	0·1425		43,740
						226·7			0·1433	
97·27	293·1	329·5	324·2	280·3	80·3	306·7	−6·4	0·1323		
95·57	301·1	339·1	335·1	292·7	80·4	317·0	−3·4	0·1298		
91·37	311·5	351·2	352·1	315·0	76·8	332·4	−4·6	0·1299		
99·05	291·4	337·9	343·9	305·9	84·5	319·8	0·4	0·1280		
109·82	299·4	350·2	355·7	313·0	93·5	329·6	−7·8	0·1283		
						321·1			0·1283	
						432·0				22,300
106·7	464·1	509·6	503·2	439·2	109·1	479·1	0·7	0·1068		
106·0	463·2	508·4	501·9	438·8	108·3	478·1	−1·1	0·1069		
106·8	464·2	510·2	499·0	436·9	108·1	477·5	3·1	0·1079		
105·4	462·5	508·3	497·0	435·2	107·7	475·7	−1·1	0·1069		
104·8	454·7	498·4	493·0	430·9	105·8	469·2	1·2	0·1081		19,840
						475·9			0·1071	
						548·6				16,570
122·4	582·2	644·7	640·8	560·8	142·5	607·1	3·5	0·0938	0·0944	
						601·5				14,740
						200·3				44,340

* Difference between effective mean temperatures of furnace and specimen.

Energy (W.)	Temperature (° C.)				$(t_2-t_1)+(t_3-t_4)$	$\frac{t_1+t_2+t_3+t_4}{4}$ or T_S	T_F-T_S	Apparent thermal conductivity K_{TS}	True thermal conductivity	Electrical conductivity σ_{TS}
	t_1	t_2	t_3	t_4						
Second run										
						9·8				101,200
						240·0				39,220
						316·7				30,640
						406·4				23,950
94·95	453·8	496·0	483·7	428·2	96·7	465·2	3·6	0·1072	0·1080	
						464·8				20,280
						581·7				15,390
						625·9				14,070
						485·0				19,330
90·54	410·1	443·2	441·1	384·8	89·5	419·8	8·1	0·1105		
91·09	410·8	443·7	441·9	385·3	89·5	420·4	8·1	0·1111		
100·00	421·9	458·3	457·5	396·5	97·4	433·6	2·5	0·1121		
						424·6			0·1140	
						431·1				22,120
78·91	352·3	377·6	376·3	331·0	70·6	359·4	−1·9	0·1221		
78·18	350·8	376·2	374·4	329·7	70·1	357·8	2·7	0·1218		
76·29	349·6	374·6	373·0	329·0	69·0	356·6	3·7	0·1208		27,210
						358·0			0·1220	
						192·4				46,620
						105·6				65,440
Third run										
						62·9				79,860
30·01	56·5	65·4	66·6	56·7	18·8	61·3	−1·9	0·1743		
25·18	53·1	60·8	61·8	53·1	16·4	57·2	7·1	0·1672		
28·26	56·1	64·8	65·8	56·3	18·2	60·7	4·5	0·1695		
28·16	56·1	64·6	65·7	56·3	17·9	60·7	1·9	0·1717		
28·27	55·2	63·7	64·6	55·1	18·0	59·6	1·1	0·1710		
28·30	54·6	63·1	64·3	54·8	18·0	59·2	1·1	0·1717		
						60·4			0·1724	
69·74	107·2	129·3	132·2	108·5	45·8	119·3	−0·9	0·1661		
67·68	110·1	131·8	134·6	111·0	45·3	121·8	2·0	0·1631		
63·08	112·1	133·3	135·8	113·0	44·0	123·5	6·5	0·1565		
61·20	106·7	126·8	129·3	107·5	41·9	117·6	6·2	0·1595		
62·95	107·3	127·3	129·6	107·5	42·1	117·7	−2·2	0·1633		
62·6	107·2	127·3	129·6	107·5	42·2	117·4	−0·5	0·1620		
61·52	109·8	129·7	131·7	109·7	41·9	120·3	3·5	0·1608		
						119·7			0·1620	
76·09	197·7	223·8	223·6	191·6	58·1	209·2	7·2	0·1429		
75·19	208·3	234·8	236·0	205·7	56·8	221·2	−2·0	0·1445		
74·65	207·7	233·9	234·8	204·5	56·5	220·1	−1·4	0·1443		
74·37	207·4	234·5	237·4	208·7	55·8	222·0	−2·2	0·1453		
68·83	203·6	229·1	231·9	205·1	52·3	217·4	1·3	0·1438		
						222·5			0·1438	
						213·9				43,590
						544·9				16,850
85·77	548·6	585·5	576·5	518·7	94·7	557·3	1·0	0·0989		
85·58	547·6	584·1	575·2	517·7	94·0	556·2	1·0	0·0994		
85·87	546·9	583·4	574·6	517·1	94·0	555·5	1·0	0·0998		
85·60	545·7	582·1	573·6	516·1	94·0	554·4	0·5	0·0994$_5$		
						555·8			0·0996	
						560·0				16,250
						646·5				13,420
						652·6				13,270
						658·6				13,130
						680·8				12,530
						733·6				11,180
						743·2				10,980
						790·0				10,030
						813·0				9,870
						811·7				9,820
						602·3				14,630
						559·1				16,240
						175·3				49,330

nergy (W.)	Temperature (° C.)				$(t_2-t_1)+(t_3-t_4)$	$\frac{t_1+t_2+t_3+t_4}{4}$ or T_S	T_F-T_S	Apparent thermal conductivity K_{TS}	True thermal conductivity	Electrical conductivity σ_{TS}
	t_1	t_2	t_3	t_4						
Fourth run										
						15·3				96,800
						198·8				45,660
73·78	220·2	247·9	250·8	223·2	55·3	235·5	−2·6	0·1457		
73·03	220·1	246·7	246·9	217·4	56·1	232·8	−0·1	0·1421		
73·32	219·9	246·2	246·3	216·9	55·7	232·3	0·0	0·1438		
72·75	226·6	254·4	256·1	227·0	56·9	241·0	1·4	0·1395		
73·06	226·2	253·5	254·7	225·8	56·2	240·0	1·5	0·1418		
72·19	226·2	253·6	254·6	225·9	56·1	240·1	1·1	0·1404		
72·52	225·9	253·2	253·8	225·1	56·0	239·5	2·4	0·1414		
						237·3			0·1425	
						551·6				16,190
76·17	561·2	593·9	585·6	534·1	84·2	568·7	−0·9	0·0988		
76·40	558·5	591·6	582·8	531·6	84·3	565·1	0·0	0·0990		
76·35	558·4	591·0	582·6	531·5	83·7	565·9	0·0	0·0996		
75·91	554·8	587·5	578·9	527·9	83·7	562·2	1·0	0·0990		
						565·5			0·0990	
75·85	627·4	663·6	656·5	603·0	89·7	634·6	0·0	0·0923	0·0923	
						643·0				13,520
75·10	695·7	737·4	729·4	669·5	101·6	708·0	2·8	0·0807		
73·80	689·5	729·7	722·5	666·6	96·1	702·1	2·9	0·0839		
74·08	686·8	727·8	722·0	667·1	95·9	700·9	2·3	0·0843		
						703·7			0·0845	
						700·8				12,040
						708·1				11,900
						771·4				10,270
76·75	753·7	807·2	804·1	746·3	111·3	777·8	1·3	0·0753		
81·45	746·6	799·4	795·6	735·8	112·6	769·3	2·1	0·0790		
81·22	748·5	801·4	798·4	735·4	113·9	771·4	0·3	0·0779		
85·01	760·4	817·1	813·3	747·3	122·7	784·5	2·4	0·0757		
						774·7			0·0776	
85·63	777·2	836·2	834·3	764·4	128·9	803·0	6·6	0·0726	0·0750	
						813·5				9,870
						792·0				10,050
						703·0				11,980
80·37	639·3	685·0	680·0	622·8	103·9	656·5	3·3	0·0844		
80·95	635·7	681·9	678·5	623·5	101·2	654·9	2·9	0·0874		
81·48	638·0	685·3	683·5	632·4	98·4	659·8	−1·4	0·0904		
						657·1			0·0890	
						665·9				12,930
						505·2				18,580
76·30	454·3	492·7	492·6	452·6	78·4	472·8	1·8	0·1063		
76·19	449·4	487·3	487·7	448·6	77·0	468·3	1·6	0·1080		
76·06	447·7	491·2	497·7	465·9	75·3	475·6	−2·5	0·1103		
75·50	459·3	504·6	510·9	477·7	78·5	461·2	4·5	0·1050		
						469·5			0·1086	
						307·4				31,840
						260·5				37,030
72·70	210·4	236·9	234·6	203·8	57·3	221·4	6·0	0·1385		
66·15	209·3	236·2	239·2	217·4	48·6	225·6	−3·0	0·1485		
65·04	204·6	229·8	230·6	207·2	48·6	218·0	1·6	0·1461		
64·10	204·4	229·8	230·5	206·7	49·2	217·8	1·3	0·1423		
61·36	215·2	236·5	236·4	213·3	44·4	214·2	−0·4	0·1484		
56·50	208·6	230·5	230·8	208·7	44·0	219·6	3·2	0·1402		
55·77	207·7	229·2	229·5	207·8	43·2	218·5	2·3	0·1410		
						219·3			0·1450	
						117·3				63,330
						14·0				100,800

Alternative method in which energy was measured by means of a water-flow calorimeter

Nickel-plated rod of Armco iron

Energy (cal.)	Temperature (° C.)			$\frac{1}{2}(t_1+t_2)$ or T_S	T_F-T_S	Apparent thermal conductivity K_{TS}	True thermal conductivity
	t_1	t_2	t_1-t_2				
3·85	32·9$_7$	22·6$_1$	10·3$_6$	27·7	2·0	0·1701	
4·32	34·6$_2$	23·3$_3$	11·2$_9$	29·0	− 0·2	0·1749	
4·51	35·9$_1$	24·0$_6$	11·8$_5$	30·0	1·4	0·1739	
4·23	34·8$_8$	23·6$_7$	11·2$_1$	29·1	− 0·9	0·1724	
4·30	35·2$_8$	23·8$_8$	11·4$_0$	29·5	− 0·9	0·1724	
				29·1			0·1744
8·09	54·8$_4$	33·3$_0$	21·5$_4$	44·1	− 0·9	0·1717	
8·33	55·6$_2$	33·8$_3$	21·7$_9$	44·7	0·3	0·1746	
				44·4			0·1736
11·05	70·6$_8$	40·9$_0$	29·7$_6$	55·8	0·4	0·1697	
11·34	71·3$_7$	41·3$_4$	30·0$_3$	56·4	0·5	0·1725	
				56·1			0·1707
20·63	122·5$_3$	65·7$_2$	56·8$_1$	94·1	7·2	0·1659	
22·00	127·8$_3$	66·2$_0$	61·6$_3$	97·0	0·7	0·1631	
21·75	126·4$_9$	65·8$_8$	60·6$_1$	96·2	1·4	0·1640	
21·57	125·9$_5$	66·1$_2$	59·8$_3$	96·0	0·0	0·1647	
22·17	129·3$_7$	67·8$_9$	61·4$_8$	98·6	1·0	0·1648	
22·28	130·9$_9$	68·9$_2$	62·0$_7$	99·9	0·5	0·1640	
				97·0			0·1640
31·79	189·7	96·9	92·8	143·3	− 2·85	0·1565	
32·94	196·7	100·1	96·6	148·4	− 6·9	0·1558	
33·65	197·0	99·3	97·7	148·2	7·1	0·1574	
32·74	191·6	97·2	94·4	143·0	4·0	0·1584	
34·62	203·3	101·9	101·4	152·6	− 2·0	0·1560	
				147·1			0·1569
47·20	284·7	137·3	147·4	211·0	1·8	0·1463	
48·46	287·0	139·0	148·0	213·0	24·0	0·1496	
46·75	284·5	137·3	147·2	210·9	− 16·0	0·1451	
45·97	280·8	135·3	145·5	208·1	− 13·5	0·1443	
46·16	281·8	136·6	145·2	209·2	− 4·5	0·1452	
				210·4			0·1460

Unplated rod of Armco iron

Diameter, 7·345 cm.; distance between temperature points, 19·4 cm.

Energy (cal.)	Temperature (° C.)			$\frac{1}{2}(t_1+t_2)$ or T_S	T_F-T_S	Apparent thermal conductivity K_{TS}	True thermal conductivity
	t_1	t_2	t_1-t_2				
7·82	59·2	37·8	21·4	48·5	− 2·15	0·1696	
8·22	61·1	38·7	22·4	49·9	− 1·7	0·1682	
8·14	59·9	38·3	21·6	49·5	4·9	0·1727	
8·54	62·0	38·9	23·1	50·5	1·1	0·1688	
8·40	59·2	36·4	22·8	47·8	− 1·0	0·1701	
8·66	59·9	36·7	23·2	48·3	− 0·9	0·1708	
8·62	60·0	36·7	23·3	48·3	− 1·2	0·1692	
8·22	57·8	35·8	22·0	46·8	0·1	0·1712	
				48·6			0·1710
15·94	98·4	54·5	43·9	76·4	− 3·2	0·1662	
15·86	97·6	54·0	43·6	75·8	− 2·5	0·1664	
16·04	98·2	54·3	43·9	76·2	− 2·0	0·1671	
14·98	91·8	51·6	40·2	71·7	4·2	0·1707	
15·21	92·6	52·1	40·5	72·4	4·4	0·1718	
14·42	90·3	50·8	39·5	70·5	− 1·3	0·1676	
14·51	89·9	50·4	39·5	70·1	− 1·3	0·1683	
14·47	90·4	50·8	39·6	70·6	− 0·9	0·1672	
				73·0			0·1684
24·75	150·2	79·9	70·3	115·0	− 1·3	0·1612	
27·68	168·2	88·1	80·1	128·2	− 3·5	0·1582	
				121·6			0·1604

DISCUSSION

Dr D. Owen said that the curve in figure 3 seemed to be linear at its ends but not in the middle. Some change appeared to take place between 400 and 500° C. The electric curve, figure 4, showed no discontinuity, however. Were similar results obtained with all specimens?

Dr L. F. Bates. I should like to ask whether any changes in the resistance due to the ferromagnetic properties of the iron were observed. I should have expected a break in the curve of conductivity with temperature, in the neighbourhood of 750° C., but perhaps the scale does not permit this to be seen. It is well known that the resistance of a ferromagnetic metal is less than that of a normal metal by a term which is proportional to the intrinsic or spontaneous magnetization. This behaviour is well established in the case of nickel and I should expect to see evidence of it here.

Author's reply. I agree with Dr Owen that there is a slight change in the slope of the thermal-conductivity, temperature curve for iron at about 400° C., although the magnitude of the change is only about half of that shown in figure 3 where the effect is exaggerated by the presence of the nickel plating.

In reply to Dr Bates: the resistance, temperature curve is typical of that of a ferromagnetic metal at temperatures below the critical point, and I hope to deal with both the thermal and electrical conductivity changes which occur within and above the critical region in a later communication.

537·312

THE CONDUCTIVITY OF METALS

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ABSTRACT. If the resistances of the pure metals are measured for the same amplitude of thermal oscillation of the atoms they show a marked periodic behaviour, being always two or three times greater for a monovalent element than for the divalent metal next to it in the periodic table. This fact is discussed on the basis of wave mechanics, and it is suggested that the periodic variation is due to a smaller effective number of free electrons in the divalent metals. Evidence in support of this view can be obtained from the optical constants of metals (§ 3) and from the increase of resistance due to foreign metals in solid solution (§ 4).

Bridgman's measurements of the change of resistance under pressure are discussed on the basis of wave mechanics (§ 5). In §§ 6 and 7 a quantitative theory is given of the behaviour under pressure of alloys and of pure metals at low temperatures, and in § 8 a qualitative explanation is given for the abnormal metals Ca and Sr, whose resistance increases under pressure.

§ 1. INTRODUCTION

In this paper we shall discuss the electrical resistance of pure metals with reference to their place in the periodic table; we shall show how the observed increase of resistance due to foreign metals in solid solution leads to a better understanding of the factors affecting the resistance of the pure metals. We shall discuss also the change of resistance of metals and alloys under pressure, both at high and low temperatures, and show that the behaviour of both normal and abnormal metals can be accounted for qualitatively, and in some cases quantitatively.

It has been known since about 1910 that the electrical resistance of a pure metal depends in some way on the thermal motion of its constituent atoms. Kammerlingh Onnes*, Nernst† and Lindemann‡, for instance, remarked that, both in the region of high and low temperatures, i.e. above and below the Debye characteristic temperature Θ, the resistance of a given pure metal was approximately proportional to its thermal energy. To account for this fact, Wien§ in 1913 developed a theory of electronic conduction, in which the mean free path of an electron was *assumed* to be inversely proportional to the square of the mean thermal displacement of the ions. Grüneisen‖, moreover, was able to account qualitatively on the basis of Wien's assumptions for the change of resistance of many metals under pressure.

* *Comm. Leiden*, **119** B, 1 (1911).
‡ *Berlin. Ber.* (1911) 316.
‖ *Verh. d. d. phys. Ges.* **15**, 186 (1913).
† *Berlin. Ber.* (1911) 306.
§ *Berlin. Ber.* (1913) 184.

The application of quantum mechanics to the problem by Bloch and others has provided a theoretical justification of Wien's assumption about the mean free path. The argument is as follows. An electron wave can pass quite freely through a perfect lattice, which therefore has infinite conductivity. A finite resistance only arises because, owing to the thermal motion of the ions or to the presence of impurities, the lattice is not perfect. At temperatures T greater than the characteristic temperature Θ, when there is no correlation between the thermal motion of one ion and that of its neighbour, one may calculate as follows the way in which the conductivity depends on temperature. If an ion is displaced from its mean position through a distance x, the amplitude of the incoherently scattered electron wave is proportional to x, and its intensity to x^2. Clearly the resistance will be proportional to the mean square of the atomic displacement, which we shall denote by $\overline{x^2}$. Now if μx is the restoring force on a displaced ion, $\frac{1}{2}\mu x^2$ is the mean value of the potential energy corresponding to a displacement in the x direction, and may thus be equated to $\frac{1}{2}kT$. We have therefore

T Θ x μ

$$\overline{x^2} = kT/\mu.$$

But if ν is the atomic frequency, $4\pi^2\nu^2 = \mu/M$, where M is the mass of an ion. Since $h\nu = k\Theta$ we have, therefore,

ν, M

$$\overline{x^2} = \frac{h^2}{4\pi^2 k}\frac{T}{M\Theta^2} \qquad \text{.....(1).}$$

To discuss the conductivity further, it is convenient to separate out the various factors as follows. Let us suppose that a metal contains N atoms per unit volume and that the current that would be produced in time δt by a field E, if there were no resistance, is

N

$$\frac{Ne^2}{m} f E\, \delta t.$$

The pure number f is equal to unity for a monovalent metal, two for a divalent metal, etc., only if the electrons are quite free—i.e. if the field of the ions is negligible. According to wave mechanics, for most real metals it is rather less than unity.

f

We write for the conductivity σ,

σ

$$\sigma = \frac{Ne^2}{m} f.\tau.$$

The quantity τ is thus the time of relaxation of the current. For a good conductor such as silver the quantity τ is of the order of magnitude of

τ

$$\frac{10^{-2}\text{ cm.}}{c} \text{ or } 3 \times 10^{-13}\text{ sec.}$$

τ is of course proportional to $\overline{x^2}$, so that

$$\tau \propto M\Theta^2/T.$$

f, on the other hand, is independent of T and of Θ.

§ 2. CONDUCTIVITIES OF PURE METALS

In table 1 are shown the measured values of the conductivities of some metals at 0° C. divided by $M\Theta^2$. The values given in column 4 therefore represent the conductivities of the metals for a value of the atomic displacement x which is the same for all the metals shown. The periodic nature of the series of values obtained is striking, in particular the large values for the alkalis and noble metals, the drop by a factor varying from 2 to 4 on passing to the divalent and trivalent metals, and the low values for the transition elements such as Fe, Ni, Pt. A periodic behaviour of this type has already been noticed by Grüneisen*, who plotted against the atomic number rather a different quantity, namely the conductivity measured at a temperature equal to $\frac{1}{2}\Theta$.

Now it is improbable that the scattering-power of an ion for given displacement—i.e. C in Bethe's equation (2) given below—and hence the mean free path will vary very much for two adjacent elements, although it is almost certainly very different for, say, K and Ag. It follows that the time of relaxation τ for given atomic displacement will vary smoothly from atom to atom, and thus that the periodic behaviour shown in table 1 is not due to τ.

It is further obvious that the periodicity in the conductivity is not simply determined by the atomic volume, since Lothar Meyer's well-known curve, in which atomic volume is plotted against atomic number, shows strong maxima for the alkali metals but no maxima at all for Cu, Ag and Au. We must therefore attribute the periodic behaviour of the conductivities to f, i.e. the factor which determines to what extent the electrons in a metal are free. The fact that f should be less for a divalent than for a monovalent metal is of course a fairly obvious consequence of the Bloch-Brillouin theory of metals, as has been realized for some time†.

One of the objects of this paper is to discuss methods by which f may be measured independently of τ (§§ 3 and 4). We shall first, however, discuss the probable values of τ from the point of view of wave mechanics.

In Bloch's treatment the state of each electron is defined by a wave function of the type

$\psi_{\mathbf{K}}$

$$\psi = e^{i(\mathbf{Kr})}\,\psi_{\mathbf{K}}(x, y, z),$$

$W(\mathbf{K})$ with energy $W(\mathbf{K})$. The conductivity has only been treated for the case in which W is a function of the modulus K of $\mathbf{K}$; in this case Bethe‡ gives the following formula for the conductivity

$$\sigma = \frac{1}{\pi^3}\, ne^2 \left(\frac{K}{C}\frac{dW}{dK}\right)^2 \frac{2\pi k}{\hbar K a_0}\,\frac{\Theta^2}{T}\,\frac{M}{m} \qquad \ldots\ldots(2),$$

n, a_0 where n is the number of electrons per atom and a_0 the radius of a hydrogen atom,

C K, dW/dK refer to the maximum energy of the Fermi distribution, and C is a

* *Handb. d. Phys.* **10**, chap. 1.

† Cf. for example, Brillouin, *Die Quantenstatistik*, 313 (Berlin 1931).

‡ *Handb. d. Phys.* **24** 2, 521 (1933).

Table 1.

	1 M	2 Θ*	3 σ†	4 $\sigma/M\Theta^2$ (arbitrary units)
Li	7	510	11·8	0·65
Be	9	1000	18	0·20
B	10·8	(1200)	—	—
Na	23	172	23·8	3·5
Mg	24	290	25	1·25
Al	27	396	40	1·0
K	39	126	15·9	2·6
Ca	40	226	23·5	1·12
Cr	52	485	6·5	0·053
Mn	55	(330)	—	—
Fe	56	453	11·2	0·1
Co	59	385	19	0·22
Ni	59	375	16·1	0·20
Cu	64	310	64·5	1·05
Zn	65	235	18·1	0·52
Rb	85	(85)	8·6	1·4
Sr	88	(140)	3·3	(0·2)
Mo	96	379	20	0·146
Rh	102	370	22	0·158
Pd	107	(230)	10·3	0·237
Ag	108	215	66·7	1·32
Cd	112	168	14·7	0·47
In	115	—	11·9	—
Sn	119	—	9·9	—
Cs	133	(68)	5·6	0·92
Ba	137	(110)	—	—
Ta	181	245	—	—
W	184	320	20·4	0·10
Ir	193	285	20·0	0·13
Pt	195	225	10·2	0·105
Au	197	175	49·0	0·82
Hg	201	97	4·4	0·23
Tl	204	96	7·1	0·37
Pb	207	90	5·2	0·30
Bi	209	110	1·3	0·051

* Θ is the characteristic temperature. The unbracketed values are taken from Landolt-Bornstein's *Tables*, 2te Erganzungsband, 1232, and from R. H. Fowler, *Statistical Mechanics*. The bracketed values are estimated from the compressibility, etc.

† σ is the conductivity in cm.$^{-1}$-Ω^{-1} at 0° C. Most of the values are taken from Grüneisen, *Handb. d. Phys.* **13**, 11.

quantity depending on the size of the metallic ion, but not on the structure of the crystal. It may further be shown that*

$$\frac{Ne^2}{m} f = nNe^2 \frac{4\pi^2}{Kh^2} \frac{dW}{dK} \qquad \text{.....(3).}$$

For free electrons, for which $W = h^2K^2/8\pi^2m$, this reduces to $f = n$.

We obtain for the time of relaxation, dividing (2) by (3),

$$\tau = \text{constant} \times \frac{1}{N} K^2 \frac{dW}{dK} \frac{M\Theta^2}{T}.$$

For free electrons $N \propto K^2$, and in general it is probable that the error is not serious if we write

$$\tau = \text{constant} \times \frac{dW}{dK} \frac{M\Theta^2}{T} \qquad \text{.....(4).}$$

For divalent metals, where the first Brillouin zone is nearly full, the assumption that W is a function only of K is certainly false, and equation (2) cannot be applied. An exact expression may however be given for f as follows. The acceleration of an electron in the state defined by $\mathbf{K}$ due to a field F in the x direction is

$$\ddot{x} = eF \frac{\partial^2 W}{\partial K_x^2} \frac{4\pi^2 m}{h^2}.$$

If N be the number of atoms per unit volume, then

$$\frac{Ne^2}{m} f = \frac{4\pi^2 e^2 m}{h^2} \frac{2N}{V} \iiint \frac{\partial^2 W}{\partial K_x^2} dK_x\, dK_y\, dK_z.$$

It is easily seen that, if W is a function of K only, this formula reduces to (3), since in this case $2N/V = \frac{4\pi}{3} K_{\max}^3$.

The formula may easily be transformed to

$$\frac{Ne^2}{m} f = \frac{4\pi^2 e^2 m}{h^2} \frac{2N}{V} \iint \frac{1}{|\text{grad } W|} \left(\frac{\partial W}{\partial K_x}\right)^2 dS \qquad \text{.....(5),}$$

integrated over the surface of the Fermi distribution. The smallness of this quantity for divalent metals is due to two causes. (1) Over a large part of the surface $\partial W/\partial K_x = 0$. Thus in figure 1 $\partial W/\partial K_x$ vanishes along AB and along CD, showing that the part of the Fermi distribution within $ABB'A'$, $CDD'C'$ is not displaced at all under the action of the field. (2) Even on parts of the surface BC, etc., $\partial W/\partial K$ is smaller than it would be for free electrons, or for a monovalent metal, especially near to B and C. H. Jones†, however, has shown that dW/dK will have, over most of the surface BC, a value nearly equal to its free electron value. The first effect, therefore, is the more important.

For divalent metals, although no exact theory of τ can be given at present, we shall assume that a fair estimate can be made by using formula (4) and by averaging dW/dK over that part of the surface of the Fermi distribution for which it does not vanish, because it is only between such points that electronic transitions can take

* See equation (5) below.

† *Proc. R.S.* (in the press).

place. Since, as we have said, dW/dK has very nearly its free-electron value over these parts of the surface, we shall be fairly safe in assuming that τ will be nearly the same for a monovalent and a divalent element which are near together in the periodic table, for given atomic displacement. *Thus we may assume that, for the values of $\sigma/M\Theta^2$ in the last column of table* 1, *the time of relaxation τ is a fairly smooth function of atomic number.*

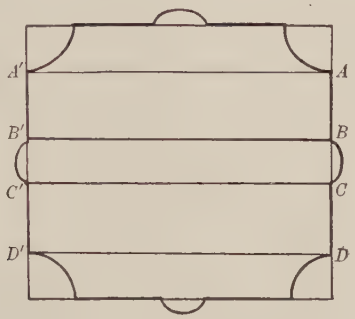

Figure 1. The Fermi surface for a divalent metal.

§ 3. OPTICAL METHODS OF DETERMINING f

The quantity f may in principle be determined by optical methods for any good conductor. If light falls on a metal, and if the period ν^{-1} of the light is small compared with the time of relaxation τ, then the current set up by the electric field of the light wave is approximately independent of τ. If the absorption of energy may be neglected altogether, so that $\tau = \infty$, the complex refractive index $\mathbf{n}$ of the metal is given by

ν

$\mathbf{n}$

$$\mathbf{n}^2 = 1 - \frac{Ne^2 f}{\pi m \nu^2}.$$

For frequencies in the visible region $\mathbf{n}$ is usually imaginary and equal to $i\kappa$, say. Thus f may be deduced if the extinction coefficient κ is measured. Unfortunately, for wave-lengths in the visible, bound electrons may have some effect on the absorption coefficient; the best way to determine f is to plot the quantity

κ

$$\frac{(1-\mathbf{n}^2)\,\pi m \nu^2}{Ne^2} \text{ or } f$$

against ν. If the values of f obtained are not independent of ν, one must extrapolate to zero frequency, or rather to the lowest frequencies consistent with the condition $\nu^{-1} \ll \tau$.

By this method, Dr Zener and the present author have obtained* the following values of f:

Metal	Li	Na	K	Rb	Ag	Au	Ni	Pt
f	0·6	1·0	0·8	0·7	1·0	0·7	0·18	0·42

* *Proc. Camb. Phil. Soc.* **30**, 262, table 1 (1934). In that paper N' is written for our Nf. The values for Ni and Pt are very uncertain, and the others are unlikely to be correct to more than 10 per cent.

§ 4. RESISTANCE OF ALLOYS

The quantity f, the free electron number, may also be determined from experiments on the resistance of alloys. If an alloy is formed in which, say, 1 per cent of a metal B is in solid solution in a metal A, then, provided that the structure is not changed, the resistance of the alloy is always higher than that of the pure metal A.

ΔR If ΔR is the difference between the specific resistance of the alloy and that of the pure metal, ΔR is found to be approximately independent of temperature, according to Matthiessen's rule. According to Nordheim*, the explanation is as follows. The

B, A field of the foreign atom B is different from that of the solvent metal A; thus extra scattering-power is caused by the difference between the fields, the extra resistance ΔR being proportional to a term of the type

$$\iiint [V_A(r) - V_B(r)]^2 \, dx dy dz \qquad \ldots\ldots(6),$$

V_A, V_B V_A and V_B being the potentials of an electron in the field of an ion of the metal A or of the metal B. Nordheim's theory does not take into account the actual distortion of the lattice by the foreign atom, and this distortion may in fact lead to a further increase in the resistance. We shall however neglect it.

For our purpose it is important to note that a term of the type (6) is symmetrical with respect to A and B; in other words, other things being equal, one atomic per cent of a metal A dissolved in B will produce the same increase of resistance as one atomic per cent of B in A. This is in fact the case for such pairs of metals as (Mg, Cd), (Cu, Au) and (Pt, Pd), as table 2 shows†.

Table 2. Change of specific resistance ΔR ($\mu\Omega$./cm^3).

Cu in Au	0·74	Au in Cu	0·6 to 0·64
Mg in Cd	0·4 to 0·45	Cd in Mg	0·5 to 0·6
Pt in Pd	0·7	Pd in Pt	0·55 to 0·6
Ag in Au	0·3 to 0·36	Au in Ag	0·3
Cu in Ag	0·4 to 0·5	Ag in Cu	0·22?

In all these cases, except Cu, Ag, the atomic volumes do not differ by more than 20 per cent.

N' On the other hand, if N' (i.e. Nf), the effective number of free electrons per unit volume, is not the same for a pair of metals, this will no longer be the case. To see that this is so, let us take the case of Ag and Mg. At very low temperatures the resistance ΔR will be the whole resistance of the alloy, by Matthiessen's rule. Let τ be the time of relaxation, at such temperatures, either for 1 per cent of Ag

* *Ann. d. Physik.* **9**, 641 (1931); cf. also Bethe, *Handb. d. Phys., loc. cit.* These calculations depend also on the assumption that W is a function of K only; but the conclusions are probably valid under the conditions stated at the end of § 2.

† Norbury, *Trans. Far. Soc.* **16**, 570 (1921).

in Mg or 1 per cent of Mg in Ag; from what has been said above it follows that the two are equal. Then for conductivity at low temperatures we have

$$1/\Delta R \propto N'_{\mathrm{Ag}}\tau \text{ for Mg in Ag;}$$

$$1/\Delta R \propto N'_{\mathrm{Mg}}\tau \text{ for Ag in Mg,}$$

where N'_{Ag}, N'_{Mg} are the numbers of electrons per unit volumes in the two metals.

Now in fact for these two alloys we find the following values*:

ΔR	ΔR
1 per cent Mg in Ag, 0·8 to 1·3;	1 per cent Ag in Mg, 3 to 3·5

and again the atomic volumes are nearly equal. We therefore deduce that the free-electron number per unit volume, f, in silver is about three times as great as for magnesium.

Another case is that of gold and cadmium*; for these the values are

1 per cent Cd in Au, 0·64 (0·35†);	1 per cent Au in Cd, 1·7 to 1·9.

In this case also the atomic volumes are almost the same.

These values, then, confirm our hypothesis that f for any monovalent metal is three or four times greater than for any divalent metal. Probably for monovalent metals the values lie between 0·7 and 1, and for divalent metals from 0·2 to 0·4, in spite of the presence of two electrons.

It is interesting to note that the two metals considered above, Ag and Mg, have about the same corrected conductivity in table 1; this must be due to the small scattering power C, equation (2), of a small ion, such as Mg, in comparison with that of a large ion, such as Ag.

Further experimental data on the change of resistance due to foreign metals in solid solution would be very interesting.

§ 5. CHANGE OF RESISTANCE UNDER PRESSURE

Bridgman‡ has found that the resistance of most metals decreases under pressure, but that for certain metals (Li, Ca, Sr, Bi) it increases. The decrease of resistance of the normal metals has been explained by Grüneisen§ in the following way. The atoms of a metal under high pressure, being closer together, are held in their positions by stronger forces, and therefore vibrate with higher frequency, than at atmospheric pressure. The atomic vibrations at given temperature are therefore smaller, and the resistance is therefore lower. If $d\nu/dp$ denotes the rate of change of atomic frequency with pressure, then, since by equation (1) the resistance is inversely proportional to the square of the frequency, we shall have

$$\frac{1}{R}\frac{dR}{dp} = -2\frac{1}{\nu}\frac{d\nu}{dp} \qquad \text{.....(7).}$$

Grüneisen has shown how $d\nu/dp$ may be calculated from the thermal expansion coefficient, and has thus been able to compare formula (7) with the results of

* Norbury, *loc. cit.*

‡ *The Physics of High Pressures* (1931).

† Earlier measurement.

§ *Verh. d. d. phys. Ges.* **15**, 186 (1913).

experiment. In table 3 we give the change of conductivity with volume obtained by dividing Bridgman's values of the pressure coefficient of resistance by the compressibility, and compare them with $-2d(\log\nu)/d(\log V)$, which, according to Grüneisen*, may be calculated from the formula

$$\frac{\alpha\,(\log\nu)}{d\,(\log V)} = -\frac{\alpha V}{\chi c_v},$$

Table 3.

Element	$-2\frac{d(\log\nu)}{d(\log V)}$ (Grüneisen)*	$-\frac{d(\log\sigma)}{d(\log V)}$ (Bridgman)	Element	$-2\frac{d(\log\nu)}{d(\log V)}$ (Grüneisen)*	$-\frac{d(\log\sigma)}{d(\log V)}$ (Bridgman)
Li	2·34	− 0·85	Mo	3·14	3·7
Na	2·50	2·84	W	3·24	4·3
K	2·68	4·0	Mn	4·84	8·3†
Rb	2·96	2·92	Fe	3·20	4·1
Cs	2·58	1·8	Co	3·74	1·76
Cu	3·92	2·7	Ni	3·76	3·35
Ag	4·80	3·6	Pd	4·46	3·75
Au	6·06	5·2	Pt	5·08	5·6
Al	4·34	2·9	Ca	—	− 1·95
Pb	5·46	6·0	Sr	—	− 6·7
Ta	3·50	2·9	Ba	—	+ 0·58

* *Loc. cit.* † Not pure.

α, c_v where α is the thermal expansion coefficient, c the specific heat and χ the compressibility. Only cubic metals are shown in the table. One sees that there is rather good agreement as regards the order of magnitude of the effect for many metals, but that the increase of conductivity is in general rather less than the predicted increase. In Cu, Sr and Li it is very much less, being converted into a decrease.

When the difference between the columns of table 3 is small, it may perhaps be attributed only to the change in the number of atoms per unit volume which results from compressing the metal; in many cases, however, it must be attributed to a variation in the effective number of free electrons per atom, i.e. to a change in f. The reason for the large decrease in Sr will be discussed below; we shall first adumbrate some consequences of this hypothesis.

§ 6. RESISTANCE OF ALLOYS UNDER PRESSURE

According to Matthiessen's rule, the extra resistance ΔR produced by a given percentage of a foreign metal such as gold in solid solution in, say, silver, is independent of temperature. From Grüneisen's theory of the pressure coefficient, it would follow that ΔR, which depends on the difference in scattering power of the two atoms, should be independent also of pressure, if, and only if, the change of resistance is entirely due to a change in ν. Thus for Cu, Ag and Au, containing foreign atoms in solid solution, one would expect ΔR to be independent of pressure, because for these metals, as table 3 shows, the change of resistance is almost entirely

* *Handb. d. Phys.* **10**, 28 (1928).

due to the change in ν. On the other hand, for lithium the effective number of free electrons must decrease under pressure, and hence the change of resistance ΔR due to foreign atoms dissolved in Li will increase under pressure, and will indeed increase more than the resistance of the normal metal; one would expect that for Li

$$\frac{d(\log \Delta R)}{d(\log V)} = -3{\cdot}2,$$

the number $-3{\cdot}2$ being the difference between the two values in table 3. Experimental evidence to test this hypothesis is not at present available, but Bridgman states that the pressure coefficient of alloys is in general less than for pure metals, as we should expect.

§ 7. PRESSURE COEFFICIENT AT LOW TEMPERATURES

At low temperatures, such that $T < \Theta$, the equation (1), $R \propto T/\Theta^2$, is no longer true; the exact formula for the conductivity due to Bloch* is

$$R = \frac{1}{\sigma} = \frac{A}{\Theta}\left(\frac{T}{\Theta}\right)^5 J\left(\frac{\Theta}{T}\right) \qquad \text{.....(8),}$$

where the function J is defined thus

$$J = \int_0^{\Theta/T} \frac{x^3\,dx}{(e^x - 1)(1 - e^{-x})}$$

and A is independent of T and Θ, but depends on the crystal structure, etc.

The function J tends to a constant value as $T \to 0$, and is proportional to $(\Theta/T)^4$ for high temperatures. It follows that, for the rate of change of conductivity with Θ, we have

$$\frac{d(\log \sigma)}{d(\log \Theta)} = \left\{\frac{\Theta/T}{J}\frac{dJ}{d(\Theta/T)} - 6\right\}.$$

This quantity may be calculated from values given by Bethe* and shown in table 4.

Table 4.

Θ/T	0	1	2	3	4	5	8	∞
$-\frac{d(\log \sigma)}{d(\log \Theta)}$	2	2·2	2·4	2·85	3·35	3·9	5·1	6

Since the change in Θ resulting from a given pressure will be approximately the same at any temperature, we should expect, for pure metals, that the pressure coefficient of resistance would increase at low temperatures; that part of it which is due to the change in Θ will be given by

$$\frac{d(\log \sigma)}{d(\log V)} = \frac{d(\log \sigma)}{d(\log \Theta)}\frac{d(\log \Theta)}{d(\log V)}.$$

That part of the pressure-change which is due to an increase in the number of free electrons, and hence of A in equation (8), will however be independent of

* Bethe, *Handb. d. Phys.* **24**, (2), 532 (1933).

temperature, and may thus be found by subtracting the two columns of table 3. Denoting this term by $d(\log A)_{d(\log V)}$ we have for the pressure coefficient of resistance at any temperature

$$\frac{d(\log \sigma)}{d(\log V)} = \frac{d(\log A)}{d(\log V)} + \frac{d(\log \sigma)}{d(\log \Theta)} \frac{d(\log \Theta)}{d(\log V)}.$$

From this formula, the pressure coefficient of resistance of a metal at any temperature T may be calculated, if the pressure coefficient at room temperature is known.

Bridgman* has measured the pressure coefficient of a series of metals at 0° C. r and at − 182·9° C. Table 5 gives the ratio r of the pressure coefficient at − 182·9° C. to that at 0° C. Column 1 gives the quantity $\frac{d(\log \sigma)}{d(\log \Theta)} \cdot \frac{d(\log \Theta)}{d(\log V)}$ at 0° C., and column 2 at − 182·9° C. Column 3 gives $\frac{d(\log A)}{d(\log V)}$, i.e. the change due to a variation in the free-electron number, obtained by subtracting columns 1 and 2 of table 3. Column 4 gives the calculated ratio r of the pressure coefficients, i.e. (1 − 3)/(2 − 3), and column 5 gives the ratio of the two pressure coefficients as measured by Bridgman. The metals are arranged in the order of increasing Θ.

Table 5.

	Θ	1	2	3	r theoretical	r experimental
Pb	90	5·5	5·75	0·5	1·04	1·0
Au	175	6·2	7·1	− 0·9	1·17	1·1
Ag	215	5·0	6·2	− 1·2	1·32	1·2
Pt	225	5·3	6·6	0·5	1·22	1·2
Pd	230	4·7	5·8	− 0·7	1·28	1·32
Cu	310	4·2	6·0	− 1·2	1·60	1·65
Ni	375	4·1	6·5	− 0·4	1·64	1·0
Mo	379	3·4	5·4	0·6	1·51	1·45
Al	396	4·8	7·9	− 1·44	1·93	2·13

Except for nickel the agreement is excellent. For the metals Fe, Ta and W (not shown in the table), however, the measured coefficients at − 182·9° C. were equal to or slightly less than those at 0° C., although the theoretical values of r are 1·6, 1·3 and 1·5 respectively. This is almost certainly due to the *Restwiderstand* of ρ_1 the metal. If the metal is impure and has a *Restwiderstand* ρ_1 independent of pressure, the measured pressure coefficient will be

$$\frac{1}{\rho + \rho_1} \frac{d\rho}{dp} \qquad \ldots\ldots(9),$$

ρ where ρ is the specific resistance of the pure metal. Bridgman in his paper gives values of ρ_{90}/ρ_{273} for the specimens used by him, and compares them with the values given in the *International Critical Tables* and repeated in table 6.

* *Proc. Amer. Acad. of Arts and Sci.* **67**, 305 (1932).

Table 6.

ρ_{90}/ρ_{273}	Pb	Au	Ag	Pt	Pd	Cu	Ni	Mo	Al	Fe	Ta	W
Bridgman	·2985	·278	·250	·249	·234	·187	·177	·158	·152	·164	·416	·313
International Critical Tables	·2953	·270	·250	·246	·329	·189	·208	·230	·189	·120	·330	·195

It will be noticed that all these metals except Fe, Ta and W were as pure or purer than those of the *International Critical Tables*; but that in Fe, Ta and W at least a third of the measured resistance must have been of the nature of *Restwiderstand*. As formula (9) shows, a *Restwiderstand* of this order of magnitude would be enough to mask the increase of $\rho^{-1}d\rho/dp$ at low temperatures.

§ 8. THE METALS WITH ABNORMAL PRESSURE COEFFICIENT

The abnormal pressure coefficients of Ca and Sr are rather easily explained. These metals are divalent and cubic. For a cubic metal the first Brillouin zone contains just two electronic states per atom, so that if the energy-gap bounding it were sufficiently large there would be no overlap into the second zone, as in figure 1, and the metal would be an insulator. To explain the abnormal pressure coefficient we must show that when the metal is compressed the energy-gap may increase faster than the total breadth of the Fermi distribution, so that the overlap, and hence f, decreases.

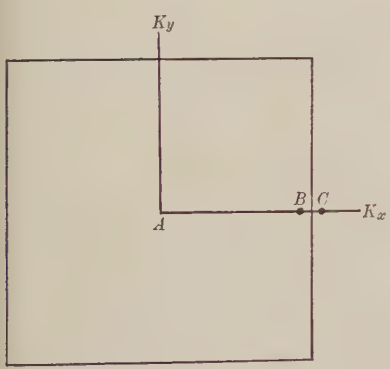

Figure 2*a*.

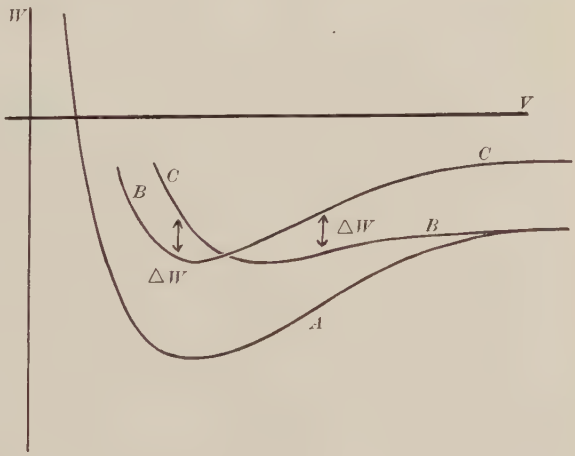

Figure 2*b*.

Figure 2*a* shows a section through the first Brillouin zone for a cubic crystal; A is the point with lowest energy, B is the point with lowest energy lying within the first zone just on a surface of energy-discontinuity, and C is the point of lowest energy in the second zone. The energy-difference between B and C is the energy-gap ΔW referred to above.

Now the wave functions corresponding to the points B and C will have nodes

either passing through the nuclei of the atoms (P functions) or mid-way between them (S functions); which of these wave functions corresponds to B, and which to C, depends on which has the lowest energy. The experiments of O'Bryan and Skinner* have shown that in magnesium the P wave functions have lower energy†, and this may well be the case also in Ca and Sr. On the other hand if one could increase the atomic volume indefinitely the energies of the S and P wave functions would tend respectively to the energy values of the lowest S and P states of the free atom. The energy of the S wave function will therefore, in the limit, be lowest. The energies of the states A, B, C will be as plotted in figure 2*b*. For some value of the volume V rather greater than the real volume of the crystal the curves B and C will cross, and ΔW will vanish.

We see, therefore, that ΔW will increase when the crystal is compressed, and that a small change of V may cause a large change in ΔW, as we require.

The discussion of non-cubic metals is more difficult, because there the pressure will change the c/a ratio as well as ΔW. It does not seem possible at present to make predictions about their behaviour. It is also difficult to explain the abnormal pressure coefficient of Li.

* *Phys. Rev.* **45**, 370 (1934). † *Ibid.* **45**, 379 (1934).

537·533

THE PHOTOELECTRONS EXPELLED FROM ELEMENTS BY CHROMIUM K RADIATIONS

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ABSTRACT. Earlier work on the magnetic spectroscopy of X-ray electrons is extended by the use of X rays of longer wave-length than those previously employed. Groups of photoelectrons ejected from targets of gold, platinum, tungsten, samarium, lanthanum, tin, silver and copper by chromium K radiations have been photographed, and their energies are tabulated and compared with the values to be expected from X-ray data. The relative intensities of the groups and other details of their appearance are briefly discussed.

§ 1. INTRODUCTION

The energies of the photoelectrons ejected from a number of elements by homogeneous X rays have already been measured in a great many cases. The present series, in which chromium K rays are employed as the primary X radiation, is an extension of earlier work in which the K radiations of copper[1,2,3], molybdenum[4] and silver[5] were used. The details of the method are given in reference (1), and the apparatus now used is that described in reference (3).

Briefly, the experiments are carried out as follows. The primary X-ray beam passes through a slit, closed airtight by a very thin aluminium or cellophane window, into an evacuated camera, where it falls upon a target composed of a narrow strip of the element under examination. The apparatus is situated in an approximately uniform magnetic field, and some of the photoelectrons emitted by the target describe approximately circular paths in the camera. A fine wedge-shaped bundle of these photoelectrons, selected by a narrow slit, is bent round by the magnetic field so that it is received by a photographic plate. The bundle splits up into a number of separate beams, corresponding to groups of photoelectrons with different velocities; these beams are made, by the application of a well-known semi-circular focusing device, to register as sharp lines on the photographic plate. The positions of these lines yield the corresponding values of rH, where r is the radius of curvature of the path of an electron in a field H perpendicular to the plane of the path. The kinetic energy K of the electron is calculated from rH by the relativity formula, values of e and e/m_0 being assumed for the purpose.

r
H
K

According to the Einstein photoelectric equation, K should be equal to $h\nu_0 - W$, where W is the work required to extract the electron from the atom and ν_0 is the frequency of the primary X rays. W also $= h\nu_c$, where ν_c is a critical absorption

W
ν_0

frequency of the element forming the target, or a frequency corresponding to a term in its X-ray spectrum, which is in most cases known from X-ray spectroscopy. The measurements therefore serve as a check both on X-ray data and on the values of the atomic constants[6].

§ 2. DESCRIPTION OF TABLES IN § 3

The substances investigated in this series were gold (79), platinum (78), tungsten (74), samarium (62), lanthanum (57), tin (50), silver (47) and copper (29). With 74, 62 and 57 thin layers of oxide were used, in other cases the metal; tin and copper were used in the form of foils, and gold, platinum and silver as thin deposits on glass or aluminium.

The results are given in tables 1 to 8. To facilitate comparison with X-ray spectral data, the kinetic energies of the photoelectrons are converted into equivalent frequencies by Planck's relation, and then divided by the Rydberg frequency. These energies, thus expressed in the customary units of νR, are denoted by ν^*/R to distinguish them from the X-ray spectroscopic values.

In calculating ν^*/R from rH, I now take provisionally $e = 4{\cdot}768 \times 10^{-10}$ e.s.u., $e/m_0 = 1{\cdot}757 \times 10^7$ e.m.u. gm.$^{-1}$, and $h = 6{\cdot}547 \times 10^{-27}$ erg.-sec. These differ appreciably from the values used in reference (3), especially for $e\,m_0$. There is however some authority for these revised figures, and it will be seen that they lead in many cases to an extraordinarily close agreement between the photoelectric results and the corresponding X-ray data.

The results tabulated for gold, tungsten, silver and copper may be compared with those given in reference (3)—bearing in mind that the rH values of Robinson, Andrews and Irons are all too high by approximately 0·08 per cent[7]; the $\nu^* R$ values are all to be reduced by 0·16 per cent on this account, and by a further 0·22 per cent to make them correspond with the values of e, $e\,m_0$ and h now adopted. The detailed comparison between the numerical values obtained in the two series is, however, deferred to a later paper, which will include additional results obtained with copper K radiation. In the present paper only the chromium K results will be considered.

In the following tables the intensity is given on a purely arbitrary scale, based only on visual estimates; 1 represents the weakest, 6 the strongest lines measured. All the photographs were taken on Schumann plates. As is well known, owing to the difficulty of accurately reproducing the extremely thin layer of emulsion, these plates tend to vary somewhat widely in sensitivity; with the Hilger Schumann plates used in the present work, it was found that although there were marked differences between different batches, very little trouble was caused by variations in sensitivity over any individual plate. The estimates of intensity are therefore not seriously vitiated by variations in the plates. On the other hand, in comparison with the plates most frequently used in the earlier work (Ilford X Ray and Process, Paget Half-Tone), the Schumann are relatively much more sensitive to slow electrons and less sensitive to swifter electrons. The relative intensities of the lines

due to the faster electron groups here recorded would have been possibly 1 or 2 units higher with the plates formerly used.

After the intensity, rH is given in gauss.-cm. then the energy ν^*/R in Rydberg units. The next column indicates the origin of the group of photoelectrons, or one or more possible origins. The fluorescent lines, representing photoelectrons of the second kind†, are indicated by the letter F. With each of these the assumed level of origin is accented in the tables, to emphasize the fact that here the photoelectron comes from an atom which is already ionized in one of its X-ray levels, and not, as is the case with the secondary cathode rays, from an atom which is initially in its normal state. The work of extraction will obviously be greater than from a normal atom, and further, as this work depends on the exact state in which the parent atom is left—i.e., on one of a great variety of possible modes of double ionization—the spectra of the fluorescent lines may be expected to be very complex. Both these expectations are realized, as may be seen from inspection of the tables, though the complexity of the fluorescent spectra is to some extent masked by the insufficiency of the resolving-power of the apparatus.

§ 3. RESULTS

In examining the tables, it is as well to remember that, as a result of the focusing method employed, the lines are in fact sharp only on their high-velocity edges, tailing off on the low-velocity side. With a close pair, then, the member of smaller energy may be obscured by the tail of its companion. Thus there are many instances where photoelectrons from the N v level can be easily measured, while those from N iv cannot be separated. Again, in other cases where resolution is just possible, there may be some doubt as to the intensity of the line of lower energy, although the position of the line may be measurable with considerable accuracy. There are many examples of this in the following tables.

The only constituents of the primary beam which need be quoted are

$$\mathrm{Cr\,K}\alpha_1 : \nu/R = 398{\cdot}8,$$
$$\mathrm{Cr\,K}\beta_1 : \nu/R = 438{\cdot}0.$$

The former of these, which is about five times as intense as the latter, is by far the more important in this work.

The X-ray data, both for levels and emission lines, have in general been taken from the last edition of Siegbahn's treatise, *Spektroskopie der Röntgenstrahlen* (1931), although in certain cases more recent data are available.

† See, in particular, references (4) and (5).

Table 1. 79, gold (metal)

Level-values, ν/R:

M I	252·2	N I	55·8	O I	7·8
M II	231·8	N II	47·3	O II, III	4·1
M III	201·9	N III	39·9	O IV, V	0
M IV	168·6	N IV	25·8		
M V	162·2	N V	24·5		
		N VI	6·2		
		N VII	5·8		

Emission lines: $M\alpha_1$ 156·4; $M\beta$ 162·4; $M\gamma$ 177·5; (M II N IV) 206·0; (M I N II, III) 212·4

Remarks	Intensity	rH	ν^*/R	Origin
Not fully resolved	(?) 2	150·4$_5$	146·5	M I : $CrK\alpha_1$ − 252·3
—	3	151·7	149·0	F : $AuM\alpha_1$ − 7·4 (N′, O′); (M II N IV) − 57·0 (N′)
—	2	154·9	155·3	F : $AuM\beta$ − 7·1 (N′, O′)
—	3	160·7	167·1	M II : $CrK\alpha_1$ − 231·7
† —	2	171·1	189·4	F : (M I O III) − 58·7 (N′) ?
—	5	174·5	197·0	M III : $CrK\alpha_1$ − 201·8
†Weak: inaccurate	0–1	183·3	217·3	F : M I − (N′ + N′) or M II − (N′ − O′)
†Weak: inaccurate	1	186·0	233·7	F : M − (N′ + N′) or M − (N′ + O′)
—	6	188·7	230·3	M IV : $CrK\alpha_1$ − 168·5
—	6	191·3	236·6	M V : $CrK\alpha_1$ − 162·2
—	1–2	204·3	269·8	M IV : $CrK\beta_1$ − 168·2
—	1–2	206·7	276·1	M V : $CrK\beta_1$ − 161·9
—	2	230·7	343·7	N I : $CrK\alpha_1$ − 55·1
—	3	233·6	352·3	N II : $CrK\alpha_1$ − 46·5
—	4	236·0	359·6	N III : $CrK\alpha_1$ − 39·2
—	5	241·0	374·9	N IV, V : $CrK\alpha_1$ − 23·9
—	3	247·2	394·3	N VI, VII, O : $CrK\alpha_1$ − 4·1
—	1	248·9$_5$	399·9	N III : $CrK\beta_1$ − 38·1
Narrow band	2	253·5	414·6	N IV, V : $CrK\beta_1$ − 23·4

† The hardest line recorded in the M emission spectrum of gold is (M I N III), with ν R 212·4—(M I O III) has never been recorded. The two faint lines with $\nu^*/R = 217·3$ and 223·7 cannot be explained as due to internal conversion of a known X-ray line. They could however perfectly well arise, in more than one way, from internal rearrangements of the atom following M I or M II excitation.

Table 2. 78, platinum (metal)

Level-values, ν/R:

M I	242·0	N I	52·3	O I	6·4
M II	221·9	N II	44·0	O II, III	3·3
M III	193·9	N III	37·3		
M IV	161·2	N IV	23·4		
M V	155·3	N V	22·3		
		N VI	4·5		
		N VII	4·3		

Emission lines: $M\alpha_1$ 151·0; $M\beta$ 156·7; $M\gamma$ 171·6; (M II N IV) 198·5; (M I N II, III) 204·7

Remarks	Intensity	rH	ν^*/R	Origin
Not accurate	1–2	139·7	126·4	F : $PtM\alpha_1$ − 24·4 (N′)
—	2–3	149·6	144·9	F : $PtM\alpha_1$ − 6·1 (N′, O′)
—	2	152·6	150·7	F : $PtM\beta$ − 6·0 (N′, O′)
—	1–2	155·4	156·3	M I : $CrK\alpha_1$ − 242·5
—	3	165·1	176·4	M II : $CrK\alpha_1$ − 222·4
—	5	177·7$_5$	204·4	M III : $CrK\alpha_1$ − 194·4
—	6	191·4$_5$	237·0	M IV : $CrK\alpha_1$ − 161·8
? Barely resolved	(?) 2	192·8	240·4	?
—	6	193·8$_5$	243·0	M V : $CrK\alpha_1$ − 155·8
Difficult : inaccurate	1	207·0	276·9	M IV : $CrK\beta_1$ 161·1
Difficult : inaccurate	1	209·2	282·9	M V : $CrK\beta_1$ − 155·1
—	1–2	231·7$_5$	346·8	N I : $CrK\alpha_1$ − 52·0
—	2–3	234·7	355·6	N II : $CrK\alpha_1$ − 43·2
—	4	236·7	361·7	N III : $CrK\alpha_1$ − 37·1
Head of a narrow band	5	241·5	376·4	N IV, V : $CrK\alpha_1$ − 22·4
—	3	247·6	395·6	N VI, VII, O : $CrK\alpha_1$ − 3·2
Possibly inaccurate; recorded on one plate only	1	249·0	400·0	N III : $CrK\beta_1$ − 38·0
	1–2	254·0	416·3	N IV, V : $CrK\beta_1$ − 21·7
	0–1	259·0	432·7	N VI, VII, O : $CrK\beta_1$ − 5·3

Table 3. 74, tungsten (oxide)

Level-values, ν/R:						
	M I	207·3	N I	43·3	O I	5·4
	M II	189·3	N II	36·0	O II, III	2·9
	M III	167·5	N III	31·0		
	M IV	137·5	N IV	18·7		
	M V	132·9	N V	17·6		
			N VI	2·3		
			N VII	2·0		

Emission lines: Mα_1 130·8; Mβ 135·1; Mγ 150·0; (M I N II, III) 176·5

Remarks	Intensity	rH	ν^*/R	Origin
—	2–3	131·2	111·5	F : WMα_1 – 19·3 (N′)
—	4	140·3	127·5	F : WMα_1 – 3·3 (N′, O′)
Doubtful : inaccurate	2	142·8	132·0	F : WMβ – 3·1 (N′, O′)
—	2	172·1	191·6	M I : CrKα_1 – 207·2
—	3	180·1	209·8	M II : CrKα_1 – 189·0
—	6	189·1	231·2	M III : CrKα_1 – 167·6
—	6	201·1	261·4	M IV : CrKα_1 – 137·4
—	6	202·9	266·1	M V : CrKα_1 – 132·7
—	1–2	204·8	271·1	M III : CrKβ_1 – 166·9
—	1–2	215·6	300·3	M IV : CrKβ_1 – 137·7
—	1–2	217·5	305·6	M V : CrKβ_1 – 132·4
—	2 –	234·7	355·6	N I : CrKα_1 – 43·2
—	2 +	236·8	362·0	N II : CrKα_1 – 36·8
—	3–4	239·1	369·0	N III : CrKα_1 – 29·8
—	5	243·2	381·7	N IV, V : CrKα_1 – 17·1
Head of a band	3	248·1	397·2	N VI, VII, O : CrKα_1 – 1·6

Table 4. 62, samarium (oxide)

Level-values, ν/R:						
	M I	127·2	N I	25·7	O I	3·0
	M II	113·8	N II	20·0	O II, III	1·9
	M III	104·9	N III	18·6		
	M IV	81·8	N IV	10·0		
	M V	79·8	N V	9·8		

Remarks	Intensity	rH	ν^*/R	Origin
—	3	$205{\cdot}2_5$	272·3	M I : CrKα_1 – 126·5
—	4	210·1	285·3	M II : CrKα_1 – 113·5
—	6	213·5	294·5	M III : CrKα_1 – 104·3
Not quite resolved	(?) 5–6	221·6	317·2	M IV : CrKα_1 – 81·6
—	5–6	222·4	319·5	M V : CrKα_1 – 79·3
Not clear : inaccurate	1–2	227·5	334·2	M III : CrKβ_1 – 103·8
—	2	236·2	360·2	? Oxygen K : CrKα_1 – 38·6†
Barely resolved	2	240·8	374·3	N I : CrKα_1 – 24·5
Barely resolved	2	$242{\cdot}9_5$	381·0	N II, III : CrKα_1 – 17·8
Barely resolved	2	$245{\cdot}8_5$	390·0	N IV, V : CrKα_1 – 8·8
Barely resolved	1	$248{\cdot}5_5$	398·6	N VI, VII, O : CrKα_1 – 0·2

† Oxygen K level, $\nu/R = 39{\cdot}3$ (Chalklin and Chalklin), or 38·8 (Thibaud). See reference (8).

Table 5. 57, lanthanum (oxide)

Level-values, ν/R:								
	L I	461·4	M I	101·0	N I	20·6	O I	3·0
	L II	434·7	M II	88·5	N II	15·0	O II, III	0·9
	L III	404·4	M III	82·5	N III	13·9		
			M IV	63·1	N IV	8·3		
			M V	61·8	N V	7·8		

Emission lines†: $L\alpha_1$ 342·6; $L\beta_1$ 371·4; $L\beta_2$ 396·6; $L\gamma_1$ 426·4

Remarks	Intensity	rH	ν^*/R	Origin
—	2	199·0	256·0	F : $LaL\alpha_1$ – 86·6 (M′)
Narrow band: not clear	2	207·1	277·2	F : $LaL\alpha_1$ – 65·4 (M′)
—	5	$214·9_5$	298·5	M I : $CrK\alpha_1$ – 100·3
Not quite resolved	(?) 3	217·8	306·5	F : $LaL\beta_1$ – 64·9 (M′)
—	4–5	219·2	310·4	M II : $CrK\alpha_1$ – 88·4
Not resolved	(?) 2	220·4	313·8	F : $LaL\beta_2$ – 82·8 (M′)
—	6	$221·2_5$	316·2	M III : $CrK\alpha_1$ – 82·6
Not resolved	(?) 5	227·1	333·1	F : $LaL\alpha_1$ – 9·5 (N′)
—	5	228·6	337·5	M IV, V : $CrK\alpha_1$ – 61·3
Difficult : inaccurate	2	234·9	356·2	M III : $CrK\beta_1$ – 81·8. Probably also fluorescent lines
—	2–3	$236·4_5$	360·9	F : $LaL\beta_1$ – 10·5 (N′)
Not quite resolved	3	$242·4_5$	379·4	N I : $CrK\alpha_1$ – 19·4
—	3	$244·2_5$	385·0	N II, III : $CrK\alpha_1$ – 13·8
—	2	246·6	392·4	N IV, V : $CrK\alpha_1$ – 6·4
Inaccurate	0–1	248·7	399·1	O : $CrK\alpha_1$ – (– 0·3)

† Some excitation of L II and L III sub-groups by $CrK\beta_1$ is to be expected, in addition to excitation by the harder components of the general primary radiation. $CrK\alpha_1$ just fails to excite the L levels of lanthanum.

In the first photographs taken with lanthanum, part of the copper backing of the chromium anticathode was exposed: as a result, the lines due to $CuK\alpha_1$ (rH = 592·7) and shown in table 5*a* were recorded:

Table 5*a*.

Remarks	Intensity	rH	ν^*/R	Origin
Not accurate	2	142·2	130·9	L I : $CuK\alpha_1$ – 461·8
—	3	156·6	158·7	L II : $CuK\alpha_1$ – 434·0
—	6	$170·8_5$	188·9	L III : $CuK\alpha_1$ – 403·8

Table 6. 50, tin (metal)

Level-values, ν/R:								
	L I	327·4	M I	63·9	N I	8·8	O I (– 1·2)	
	L II	304·9	M II	54·4	N II, III	5·3		
	L III	288·1	M III	51·2	N IV	0·6		
			M IV	35·1	N V	0·4		
			M V	34·4				

Emission lines: $L\alpha_1$ 253·7; $L\beta_1$ 269·8; $L\beta_2$ 287·7; $L\gamma_1$ 304·3

Remarks	Intensity	rH	ν^*/R	Origin
Inaccurate	1–2	103·5	69·4	L I : $CrK\alpha_1$ – 329·4
—	2–3	119·7	92·8	L II : $CrK\alpha_1$ – 306·0
—	5	129·9	109·3	L III : $CrK\alpha_1$ – 289·5
Not clear	1–2	151·6	148·8	L III : $CrK\beta_1$ 289·2
Not sharp: head of a band	3–4	174·9	197·9	F : $SnL\alpha_1$ – 55·8 (M III′)
Head of a band	6	$182·3_5$	215·1	F : $SnL\alpha_1$ – 38·6 (M IV, V′)
—	5	189·3	231·7	F : $SnL\beta_1$ – 38·1 (M IV, V′)
Head of a band	3–4	$197·3_5$	251·8	F : $SnL\beta_2$ 35·9 (M IV, V′); $L\alpha_1$ – 1·9 (N′, O′)
Doubtful line	0–1	198·6	255·0	F : $SnL\beta_1$ – 14·8 (N′)
Broad band	1–2	203·7	268·2	F : $SnL\beta_1$ – 1·6 (N′, O′)
—	5	$227·6_5$	334·7	M I : $CrK\alpha_1$ – 64·1
Not fully resolved	(?) 5	230·8	344·0	M II : $CrK\alpha_1$ – 54·8
—	5–6	$231·8_5$	347·1	M III : $CrK\alpha_1$ – 51·7
Double	4	237·4	363·8	M IV, V : $CrK\alpha_1$ – 35·0
Band: not sharp	1–2	247·0	393·7	N, O : $CrK\alpha_1$ – 5·1

Table 7. 47, silver (metal)

Level-values, ν/R: L I 282·0, L II 261·3, L III 248·6; M I 54·7, M II 46·1, M III 43·7, M IV 29·2, M V 28·8; N I 8·7, N II, III 5·9, N IV 2·1, N V 1·9

Emission lines: $L\alpha_1$ 219·8; $L\beta_1$ 232·1; $L\beta_2$ 246·7; $L\gamma_1$ 259·2

Remarks	Intensity	rH	ν^*/R	Origin
—	2–3	135·3	118·6	L I : $CrK\alpha_1$ − 280·2
—	4	146·7	139·3	L II : $CrK\alpha_1$ − 259·5
—	6	153·3$_5$	152·2	L III : $CrK\alpha_1$ − 246·6
—	2–3	164·7$_5$	175·7	F : $AgL\alpha_1$ − 44·1 (M III′); $AgL\beta_1$ − 56·4 (M I′)
Not resolved	(?) 2	169·6	186·1	F : $AgL\beta_1$ − 46·0 (M II′)
Not resolved	(?) 3–4	170·7	188·5	F : $AgL\beta_1$ − 43·6 (M III′)
Head of a complex band	6	171·5	190·3	F : $AgL\alpha_1$ − 29·5 (M IV, V′)
Doubtful line	0–1	171·8$_5$	191·1	F : $AgL\alpha_1$ − 28·7 (M V′)
—	4	177·2	203·1	F : $AgL\beta_1$ − 29·0 (M IV, V′)
Head of a broad band	2	184·2	219·5	F : $AgL\beta_2$ − 28·2 (M IV, V′); $L\alpha_1$ − 0·3 (N′)
—	1–2	186·1$_5$	224·1	F : $AgL\beta_1$ − 8·0 (N′)
—	1–2	189·4	232·0	F : $AgL\beta_1$ − 0·1 (N′, O′)
—	4	231·7	346·7	M I : $CrK\alpha_1$ − 52·1
Not quite resolved	(?) 5	234·6	355·3	M II : $CrK\alpha_1$ − 43·5
—	5	235·3$_5$	357·6	M III : $CrK\alpha_1$ − 41·2
—	3–4	240·2$_5$	372·6	M IV, V : $CrK\alpha_1$ − 26·2
Head of a narrow band	2–3	247·8	396·2	N : $CrK\alpha_1$ − 2·6
—	1	252·9	412·6	M IV, V : $CrK\beta_1$ − 25·4

Table 8. 29, copper (metal)†

Level-values, ν/R: L I 81·0, L II 70·3, L III 68·9; M I 8·9, M II, III 5·7, M IV, V 0·4

Remarks	Intensity	rH	ν^*/R	Origin
—	6	222·0	318·3	L I : $CrK\alpha_1$ − 80·5
—	6 –	226·1	330·2	L II, III : $CrK\alpha_1$ − 68·6
Inaccurate: band with diffuse edge	2–3	246·2	391·2	M : $CrK\alpha_1$ − 7·6

† The experiments with copper were done under rather unfavourable conditions, and it was not possible to measure the M groups accurately.

§ 4. AGREEMENT WITH X-RAY DATA

It will be seen that with the values now adopted for the atomic constants there is in general good agreement between the photoelectric and crystal values of the X-ray terms, and that on the whole the agreement is best where the experimental conditions are most favourable. It will be obvious that when the photoelectric equation is used in this way to obtain term values, the experimental errors are greatest for the terms of lowest (negative) energy. For example, with a given element the M terms can be obtained much more accurately than the N.

Perhaps the most striking discrepancies are those in tables 6 and 7, for tin and silver. Although the consideration of the general question of numerical agreement is deferred to a later paper, it may be mentioned here that there is strong evidence that Siegbahn's values for the L and M levels of these two metals require fairly drastic revision[9,10]. It may therefore reasonably be assumed that part of the observed discrepancies is to be ascribed to inaccuracies in the X-ray data. It should however be pointed out at the same time that in some cases the photoelectric values are in better agreement with Siegbahn's summary tables than they are with recent direct determinations, for instance those of the gold and platinum M levels.

§ 5. RESOLVING POWER OF MAGNETIC SPECTROMETER

There are a number of points of general interest which emerge from the examination of these magnetic spectra. In the first place, the power and flexibility of the method are well illustrated by the way in which the M and N levels have been separated. All five M levels have been separated even for samarium (62). For lighter elements it is naturally not possible to separate M IV and M V, but M I, M II, M III, M V are separable for an element as light as silver (47). The spin doublets N IV, V and N VI, VII—the latter of which overlaps the O levels—are not resolved, but there is very fair separation of the other N levels right down to lanthanum (57).

This relatively high resolution is mainly due to the use of a fairly soft primary X radiation. Theoretically almost any degree of resolution is attainable by the use of sufficiently soft X rays, but unfortunately the more serious of the technical difficulties of these experiments increase rather rapidly with the wave-length of the primary X rays. Even with chromium K radiations the absorption is fairly high* and the photoelectrons are of such low energy that they are somewhat ineffective photographically. It seems probable that the work cannot be extended to much longer wave-lengths without some radical alterations in the apparatus and methods.

§ 6. RELATIVE INTENSITIES

The relative intensities of the lines are of special interest. The variation with the primary X-ray frequency of the relative intensities of the three lines in the L group and the five lines in the M group has already been discussed in some detail in references (1), (2), (4) and (5). The outstanding feature of the results is as follows: it is well known that when the frequency of the primary radiation is only a little higher than the highest critical absorption frequency of a group of levels, then the greater part of the absorption by that group is associated with the levels of larger k, according to the old n_k notation for the levels. As the frequency of the X radiation increases, the levels of smaller k play a relatively greater part in the total absorption of the group, and finally at high frequencies their effect is predominant. The present work throws no new light on the general problem of absorption, but it very

* The intensity of a beam of chromium Kα radiation is reduced to less than half value by absorption in a sheet of aluminium 0·02 mm. thick.

satisfactorily confirms and extends the information already obtained by this method at higher frequencies. The regularity with which these changes in relative intensity occur will be seen at once if the values listed above are compared with those shown in the diagram of reference (5), page 108.

§ 7. GENERAL

There are other points of interest in the appearance of the lines of the magnetic spectra. The line due to the conversion of the $CrK\beta_1$ line in a given level is always less clearly defined than that due to the conversion of $CrK\alpha_1$ in the same level. Comparison is rather difficult, as the former is always so much less intense than the latter, but the relative lack of sharpness of the lines due to $K\beta_1$ is noticeable on a great many of the plates. Such a difference is readily to be explained by the difference between the natural widths of the $K\alpha_1$ and $K\beta_1$ lines, but it is rather surprising that it should show up so prominently. It is also found that there are differences in sharpness between the lines corresponding to different levels, even when the lines are due to the same primary X radiation. It need hardly be said that these differences in the character of the lines are perceptible rather than measurable, and that they would be obscured by a very slight laxity in the control of the magnetic field during an exposure. They are however sufficiently marked to force themselves upon the attention of anyone habitually engaged in the measurement of the lines.

It is difficult to form a precise opinion upon the sharpness or otherwise of a set of lines, but the photographs as a whole leave an impression very much in accordance with the conclusion reached by Coster[11] as a result of his extensive and close study of X-ray emission spectra—namely that those levels are least sharply defined which have the greatest values for the ratio of their quantum numbers n/k. In terms of the old quasi-mechanical atom model, those levels are most diffuse which correspond to the orbits of greatest eccentricity. Thus, M v is sharper than M i.

This question of the finite width of the X-ray lines and of the indefiniteness of the levels requires further attention, for it may appreciably affect the level-values measured by the photoelectric method when the accuracy is being pushed to the limit. The measurements are, with the semicircular focusing method, made on the high-velocity edges of the lines; what is measured is therefore something lying between the average and the maximum velocity of the electrons constituting the group, and depending to some extent upon the distribution of velocity and upon the peculiarities of the photographic plate. The resulting error is probably not large—the effect of the finite width of the chromium $K\alpha_1$ line is not likely to be more than about $0{\cdot}1\ \nu/R$—but it is well to bear in mind that the photoelectric method, as used here, tends to give something approximating to a lower limit for an X-ray level.

§ 8. ACKNOWLEDGMENTS

In the course of this work it was necessary to take over eighty photographs of magnetic spectra, many of them being exposures of several hours' duration. I have pleasure in acknowledging my indebtedness to the Department of Scientific

and Industrial Research for a grant which enabled me to employ a full-time assistant and so to complete the work. I am also indebted to my assistant, Mr H. A. Nancarrow, M.Sc., for his share in the work; to the Research Department of the Metropolitan-Vickers Electrical Co. Ltd. for kindness in supplying me with chromium-plated copper sheets for use as anticathodes; and to Prof. E. H. Lamb for supplies of liquid air.

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A 5-KILOWATT X-RAY GENERATOR WITH A SPINNING TARGET

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ABSTRACT. A considerable increase in power can be obtained by moving the target of an X-ray generator. A detailed account is given of the design and construction of a successful machine in which the target is a water-cooled rotating disc.

§ 1. INTRODUCTION

This paper forms a sequel to three papers by A. Muller* in which he examines mathematically the power input limit of an X-ray generator, in the first paper for a circular focus on a stationary target, in the second for a circular focus on a moving target, and in the third for a line focus on both a stationary and a moving target.

A very great increase in the limit of power input is obtained by making the target in the form of a rotating disc. This has been done previously in sealed-off tubes, and large inputs have been obtained for a very short time after which the target has to be allowed to cool, mainly by radiation. If a large input is required continuously the target must be water-cooled; this means that the shaft must be brought out through a stuffing-box and that the vacuum must be maintained by continuous pumping.

For the machine described here Muller's formula gives 40 kw. as the limit of input power; this is with a line focus 1 cm. long and 1 mm. wide, the copper of the target being allowed to reach its melting-point. Cracking and roughening of the target, resulting in a decrease in X-ray intensity, occur well below the melting-point, and if the temperature-rise is limited to 300° C. the power-input limit is then 14 kw. If allowance is made for accidental local concentration of the cathode stream, due to hot spots on the filament or to imperfect focusing, the limit is considerably reduced still further. The machine works normally at 5 kw. and, at this power, has run for about 1000 hours of routine work during the past two years in addition to giving a very satisfactory performance during the experimental period.

The detailed description given is intended to show how a successful machine has been constructed. It is hoped that anyone interested and having some experience of design to suit the resources of a small workshop will find this description helpful in building a similar machine.

* *Proc. R.S.* A, **117**, 30 (1927); **125**, 507 (1929); **132**, 646 (1931).

§ 2. OUTLINE OF DESIGN

The mechanical problem is to rotate a water-cooled copper disc in a vacuum while a stream of cathode rays is directed on to it from a suitably arranged cathode, the X-rays generated being allowed to emerge from a window with as little absorption as possible.

The disc is hollow and is mounted on the end of a hollow shaft which runs in ball bearings, passes out through a stuffing-box, and is driven by a belt and pulley while water circulates through the shaft and the inside of the disc.

The enclosure in which the disc runs is exhausted by a fast diffusion pump to a pressure at which the emission from a tungsten filament can be used as the source of cathode rays. The cathode is insulated and connected to a source of current at high tension.

The whole is mounted conveniently and rigidly on the wall of the room and provision is made for standing spectrometers in front of the window. Safety devices are provided to safeguard both the apparatus and the operator from accident or foolishness.

§ 3. THE MOUNTING OF THE TARGET DISC

Figure 1 shows a cross-section of the body of the machine with the target disc in place in its ball bearings. The body is in two parts, both of brass, one, 26, of which carries the ball bearings and vacuum stuffing, while the other, 2, forms the enclosure in which the disc rotates. The disc is built up of two parts, 3 machined from a solid piece of copper, and the back plate 6 of brass which is fitted and hard-soldered into 3; this has a boss 31 fitting into the part 30 which is clamped to the inner race of the ball bearing 9 by means of the nut 10. The outer race of this ball bearing fits a recess in 26 and is held in by the ring 8 and three small screws. A small grub screw 7 passing through 30, engages a slot in the boss 31 and prevents relative rotation of 31 in 30 while permitting the disc and its shaft to be withdrawn and replaced by another of steel or of copper coated with another metal whose characteristic radiation is required. The second ball bearing 21 at the other end is mounted in a similar way on the part 25 to which it is clamped by the ring 22; 25 is secured to the shaft by means of the grub screw 20, a hole being provided in 26 through which a screw-driver can be passed for this purpose. The outer race of this ball bearing is held in a recess in 24 by the ring 23, both 23 and 24 being fixed to 26 by three screws.

The hollow steel shaft 27 is hard-soldered into the boss 31 on the back plate 6; a smaller tube 28 is mounted concentrically in the baffle plate 4 which is carried on a perforated ring 5 from the inner surface of the back plate 6. The other end of 28 passes through a bush in the end of 27 to which it is soft-soldered. The water cooling the disc enters through the fitting shown in figure 3, passes up the smaller tube 28, and over the baffle plate, impinges on the inner surface of the copper 3,

and passes thence down the annular space between 27 and 28 and to waste through three holes near the end of 27.

The stuffing which renders the two glands in figure 3 water-tight is asbestos-graphite steam-gland packing.

The hard-soldering of the parts of the disc is somewhat difficult, for it must be absolutely water-tight and if water-tightness is not attained after one or two attempts the copper itself becomes spongy and leaks. A good test for water-tightness is to fill the disc with water and to run it at high speed while a polished surface is held to catch and detect any minute drops thrown off. If this test is passed, the joint is good.

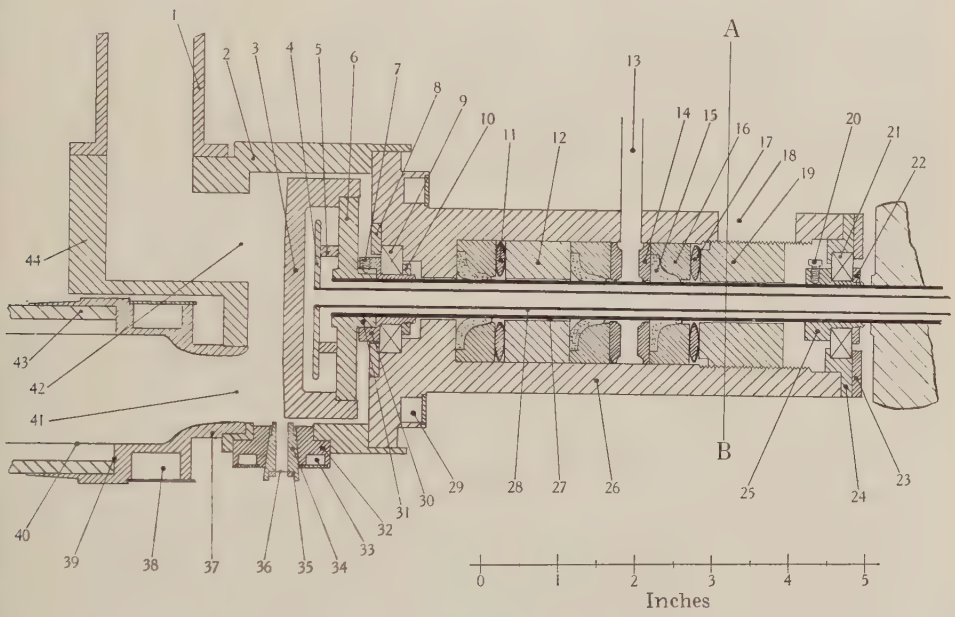


Figure 1.

§ 4. THE CASE AND WINDOW

The case 2 with the cubical boss 44 to which is attached the connexion 1 for the pump is machined out of a solid piece of brass.

Holes 41 and 42 are bored to take the support for the cathode 37 and to form the passage leading to the pump. A recess in 2 fits a flange on 26; the two parts are held together by six screws and the joint is made vacuum-tight with soft wax or, better, with apiezon plasticene. An end view is shown in figure 2, in which the parts are numbered to correspond with figure 1.

The part 37 which carries the glass tube supporting the cathode is made of steel and is fitted and soft-soldered into the hole 41 in 2. The glass tube 43 fits loosely in the open end and rests on the shoulder 39; a steel tube 40 protrudes some two or three inches and serves to remove electrical stress from the glass in

the neighbourhood of the joint. A plug 32 is soft-soldered into a recess in the side of 2 and into this fits the conical plug 34 which carries the thin aluminium window 36. This rests in a recess in 34 and is held in place with a ring 35 and a small quantity of sealing-wax, care being taken that no wax, which would seriously absorb X-rays, is allowed on either face of the window. The axis of the plug 34 lies in the plane of the disc which is slightly bevelled so as to allow X-rays from the whole length of the focus to pass through the window. The focus is formed along a radius of the disc and its foreshortened appearance as seen from the window is approximately circular.

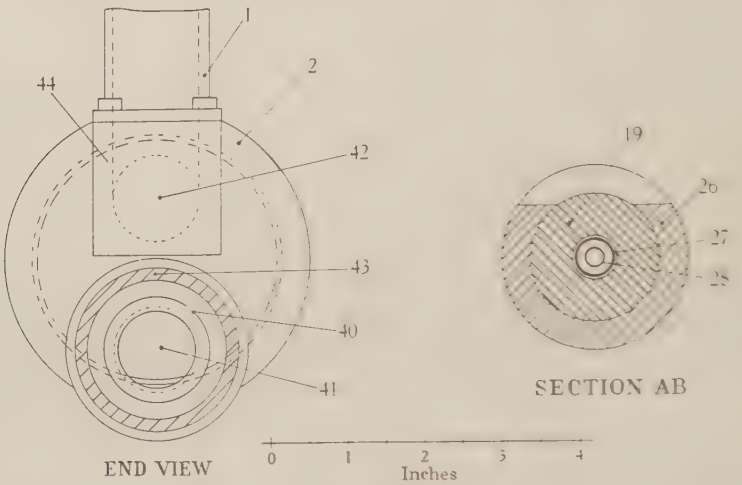


Figure 2.

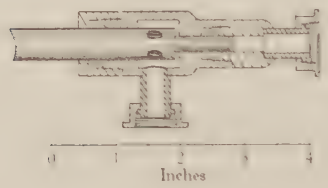


Figure 3.

The aluminium is punched with a cork-borer from a selected portion, free from pin-holes, of foil 0·001 in. thick. This absorbs about 19 per cent of copper radiation. Considerably less is absorbed by a window of beryllium, a small piece of which was distilled on to carbon in a cathode-ray furnace and subsequently ground to a suitable thickness. For some purposes, when a long collimating-tube is in use, this tube can be attached to the machine and a window of thin cellophane can be mounted on its end; in this way the absorption of the air is eliminated and the absorption of the window decreased. Secondary cathode rays must not be allowed to fall on the cellophane; if they do so a small magnet can often be arranged to deflect them.

Water cooling of the joint between 2 and 26, and of the window, is provided by channels 29 and 33 closed by rings fitted and soldered in recesses. Cooling of 37 is by water circulating in the channel 38 which is closed by a brass tube. Small nozzles made with slight swellings at their ends to hold rubber tubing firmly are fitted at convenient places. These should be hard-soldered in place so that they do not break off when the rubber tubing is pulled off.

§ 5. THE VACUUM STUFFING

The stuffing which fills the greater part of the body 26 is of special design. A hat-leather 15 fitting the shaft is compressed on to the shaft by means of a specially moulded rubber ring 16 which fits inside 26. Three stages are employed with an exhaust 13 communicating with the perforated ring 14 between the outer two.

All three stages are compressed together by the screw 19 which is turned by means of a tommy bar inserted into holes through the opening 18 cut in the side of 26. The force is communicated from the outer to the inner stages by the ring 14 and the distance piece 12.

The rings 17 and 11 are of lenticular cross section so that their area of contact with the rubber rings increases as the screw is tightened. The rubber is distorted and pressed against the inside of 26 and against the inner edges of the hat-leathers; in this way a compression which is readily adjustable is obtained, for the volume elasticity of rubber is very large, and if no space were left for distortion the pressure would rise very rapidly and adjustment would be more difficult. It is well to run the machine with the stuffing as loose as is consistent with a good vacuum. The hat-leathers are exhausted under molten vaseline in which they are allowed to cool; further vaseline is applied to the shaft as the stuffing is assembled. The rubber rings were made in a steel mould of appropriate shape to fit the hat-leathers.

In an earlier machine, which was quite successful, ordinary hemp in two stuffing boxes was used; the space between the boxes was exhausted and oil was allowed to suck through the outer stuffing from a reservoir into a detachable bulb from which it had to be transferred back to the reservoir from time to time. This return could be effected by means of a pump and a very satisfactory seal might be made in this way.

§ 6. THE CATHODE

A line focus is used so as to distribute the heat further; it is obtained from a hot filament with a shield having a slot in it. The support for the filament is shown in figure 4; it is made of steel and is seen in parts at the top of the figure and assembled at the bottom.

The rod 50 screws into 52 at one end and into 61 at the other. A screwed extension of 52 passes through the washer 53 and into the end of the conical part 55; this cone fits the conical hole in 54. When the rod and 61 are screwed up tight, the parts are held rigidly together and at the same time 54 is held firmly in the end of the main supporting tube 51. Mica insulation is wrapped round 55 and a mica

washer is inserted between 52 and 53. A projection 56 on the other end of 55 carries one clamp for the filament, and the arrangement of this clamp is shown in detail and enlarged in figure 5. A small plug fits a hole in 56 and has in it a small hole which fits the filament, a portion of this plug being filed away to permit the entry of the flat end of a screw which grips the filament securely. The other clamp for the filament is carried on the projection 58 on the end of 57; 57 and 58 are machined out of a solid piece which screws on to the front of 54.

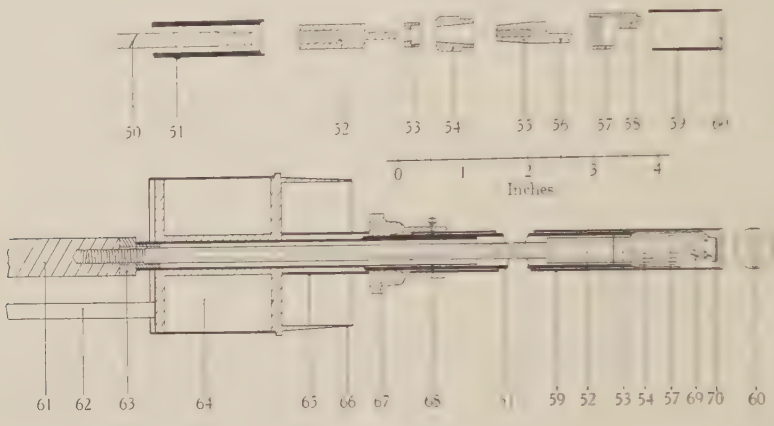
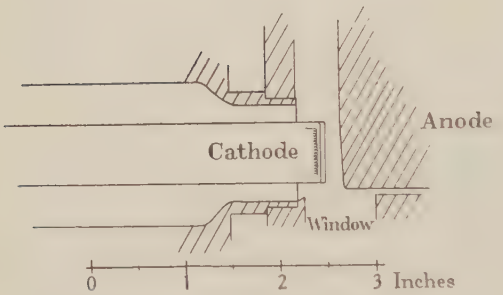
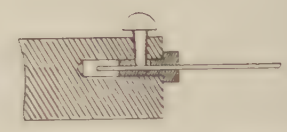


Figure 4.



Relative positions of cathode and anode.



Detail of filament-clamp (enlarged).

Figure 5.

This piece has, of course, to be screwed on while 55 is but loosely inserted in 54; the clamping portions 56 and 58 are then adjusted to be parallel and the rod 50 is tightened up by means of the nut 61.

Focusing is obtained by means of a slotted plate 60 made of the same metal as the disc in use and carried on the outer tube 59 which slides over 51 and is secured in position, with the slot 2 mm. in front of the filament, by means of the screw 68 passing through the fitting 67. The other end of this part 67 screws on to the split end of 51 and causes it to grip the smaller tube soldered into the end piece 64. This enables the distance from the slot to the surface of the target to be adjusted

to 4 mm. When this adjustment has been made, a distance piece 65 of the correct length is inserted to prevent slipping as the nut 61 is tightened. A bush of red fibre between 61 and the end of the tube completes the insulation of one of the electrodes; the electrical circuit is then completed through clamps on the end of 61 and the water pipes 62 which lead water into a channel surrounding 64. A tubular extension 66 fits loosely over the end of the glass tube which supports the cathode.

§ 7. THE FILAMENT

The filament is of tungsten wire 0·3 mm. in diameter, 22 turns of which are wound on a mandrel 1 mm. in diameter. This requires a heating current of 8·2 A. at 14 V. for an emission of 150 mA. The winding-machine is a piece of 1 mm. steel wire soldered in the tip of a 2 B.A. screw running in a nut that can be clamped in a block of brass; the tungsten is fed through a small hole in an extension from the block, and as a handle soldered to the head of the screw is turned the wire winds on the steel wire which is fed forward at the pitch of the screw. In this way a uniform spacing is obtained which avoids local over-heating. The ends of the tungsten are then bent at right angles and inserted into the clamps.

The filament should be mounted so that it is parallel to the slot, centrally placed, and 2 mm. behind the front edge of the slot; this distance is important, if it is too small the focus will not be sharp and there is risk of fusing the edges of the slot. The emission of 150 mA. will be obtained with a heating-current less than normal. If the distance is too large the heating-current has to be raised above normal to obtain the emission, and the filament will rapidly evaporate and cover the target with a deposit of tungsten.

The distance from the slot to the target should be 4 mm.; if this distance is too small, sparking will occur and it will not be possible to raise the voltage to the normal running value of 30 kV. If it is too large, the machine will generate high-frequency oscillations and behave very much as if it were leaking. After inserting a new filament and pumping to a good vacuum—10^{-4} mm. of mercury—the filament should be run at normal current for 10 minutes with the disc rotating but without the high tension.

§ 8. ASSEMBLY AND MOUNTING

The generator is shown assembled in figure 6. The cathode is supported on two glass tubes, the joints being made with short lengths 80, 82, 84 of electro-deposited rubber cycle tube; if these are washed with alcohol as they are put on they form a very good vacuum joint. Such joints have been used successfully in similar apparatus since 1929. The tube 81, supported on the flange 83 between the two glass tubes, is intended to reduce the potential-gradient; it may not, however, really be necessary for this purpose, but it is useful for centering the cathode by sliding the glass tubes on the flange.

The generator is clamped in a large plummer block bolted to a girder which is fixed to the wall. The girder also carries an adjustable table and a Leybold four-

stage mercury diffusion pump. A closed steel tube made by boring a solid bar is mounted inside the main pipe from the pump; solid carbon dioxide is packed into this and mercury vapour is thereby prevented from getting into the machine.

A Pirani gauge mounted on the top of 2 (fig. 6) is used as an indicator of leaks; this was supplied by courtesy of the Research Laboratories of the General Electric Company. The disc is driven at 2000 r.p.m. by a pulley 86 and belt from a 1-h.p. motor which is separately mounted on the wall. Next to the pulley is a centrifugal switch 87 which is connected by two slip-rings and brushes in circuit with the operating coil of the main contactor, so that if the disc should stop for any reason the power is switched off.

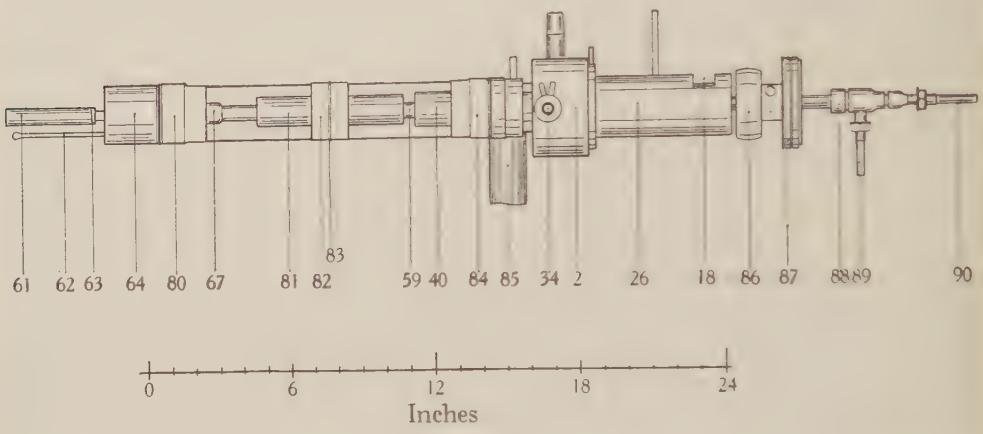


Figure 6.

88, 89, 90 are the water connexions for cooling the disc already described (p. 704 and fig. 3). From the outlet pipe a small branch pipe leads up to one of the mercury switches on the wall. These switches are operated by the pressure of the water as it emerges from a slightly constricted outlet in each water circuit and they switch off the power if the water fails. A small contactor and ammeter are in circuit with the electric heater of the diffusion pump; and the operating coil of this contactor is in circuit with a mercury switch operated by the cooling-water of the pump.

A large conical-ended shield completely encloses the cathode and its supporting tubes. The water and electrical connexions are led to the end of the cathode through a brass tube supported along the axis of the shield from a large paxolin column 4 ft. long and 5 in. in diameter; on this are wound two 30-ft. lengths of $\frac{1}{4}$-in. rubber tube to provide insulation for the water cooling the cathode. This rubber must be kept dry and should be protected from the effects of ozone with a coat of varnish applied after it has been wound on the paxolin. Shellac can be used but it is liable to crack, and then the rubber is attacked by ozone and very soon breaks down. A more flexible varnish can be obtained from insulator-manufacturers and is very much better for the purpose.

§ 9. ELECTRICAL CIRCUIT

The current of 150 mA. at 30 kV. supplied to the generator is obtained from a 5-kw., centre-tapped transformer specially made by Schall and Son, Ltd., and two rectifying valves giving biphase rectification. The valves must be capable of withstanding a back voltage double the working peak value and of passing a current of 150 mA. continuously; valves having the necessary characteristics can be obtained commercially.

The body of the generator is connected to earth and the current returns to the centre tapping of the transformer through a milliammeter. A high resistance of 10 MΩ. in series with a milliammeter reading up to 5 mA. is connected from the

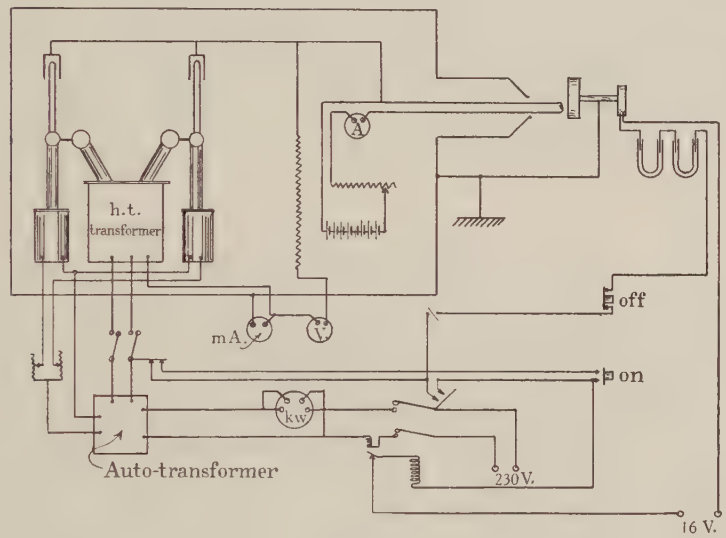


Figure 7.

high-tension terminal of the generator to the centre tapping and indicates the voltage. The input is controlled by means of variable tappings on an autotransformer and a variable series resistance.

A contactor operating on 16 V. by a push button connects the 230-V. supply to an autotransformer, and this at once heats the valve filaments through two transformers with insulated low-tension secondary windings. A manually operated switch is then used to turn on the high-tension transformer: a back contact on this switch wired in series with the "on" button ensures that the valve filaments shall be hot before the h.t. can be turned on. A series of safety contacts in the operating circuit of the contactor renders the electrical apparatus as fool-proof as possible. The centrifugal switch and the water-operated mercury switches have been mentioned; in addition there is a contact on the door of the enclosure protecting the transformers and valves.

§ 10. ACKNOWLEDGMENTS

The machine described was designed and constructed in collaboration with Dr Alex Muller, the assistant director of the Davy Faraday Laboratory, where the work has, for the greater part, been carried out. Thanks are due to Mr H. Smith who has collaborated throughout in the construction and testing of the machine and is now responsible for running and maintenance; to the workshop of the Royal Institution and in particular to the head of this workshop, Mr C. H. Jenkinson, for making the machine and for many suggestions connected with the mechanical design; to Mr Mitcham, for help in setting up the electrical equipment; and to the Research Laboratories of the General Electric Company for their continued friendly interest and hospitality for several months during the rebuilding of the Royal Institution. Finally, the author acknowledges his indebtedness to the director and managers of the Royal Institution for their interest in the work.

537·531

THE ESTIMATION OF SMALL DIFFERENCES IN X-RAY WAVE-LENGTHS BY THE POWDER METHOD

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Communicated by Prof. W. L. Bragg, February 28, 1934. *Read June* 1, 1934.

ABSTRACT. It has been found possible by the use of a microphotometer to determine accurately the positions of lines at high angles of reflection on a powder photograph. With a powder photograph of clear colourless quartz taken with copper K_α radiation, the distance apart of the two component lines of a well-resolved doublet was measured to within 0·0002 cm. The measurements were then corrected for systematic errors—eccentricity of specimen, absorption of the radiation in the specimen, and divergence of the X-ray beam. The wave-length difference $(\lambda_2 - \lambda_1)$ was finally calculated in terms of the given wave-length λ_1. The value of $(\lambda_2 - \lambda_1)$ for copper K_α radiation is given as 3·833 X.

§ 1. INTRODUCTION

During a study of the use of quartz for the calibration of X-ray powder cameras* it was found that the K_α doublets at high angles of reflection had very sharp peaks. The most highly resolved doublets are so far apart that their peaks are not likely to be displaced as a result of overlapping of their bases. This can be seen from figure 1 *a*, which is a photometric record† for the doublet for which $h, k, l = 2, 3, 4$, readings being taken at intervals of 0·01 cm. From a detailed survey of the summits of these lines, with readings at intervals of 0·001 cm., figure 1 *b*, it appears that the position of the peaks can be measured to $\pm$ 0·0001 cm.

From such measurements it should be possible to make accurate estimates of small differences of wave-length. An interesting problem is the measurement of the difference between the copper K_{α_2} and K_{α_1} wave-lengths in terms of the given copper K_{α_1} wave-length.

§ 2. EXPERIMENTAL MEASUREMENTS

Special precautions were taken to find the positions of the peaks. Figures 1 *b*, 2 *a* and 2 *b* show typical curves obtained from photometer readings taken at intervals of 0·001 cm. near the summits of the 2, 3, 4 doublet on both sides of the film. The exact position of maximum density cannot be read off the curve directly because the top is rounded. In order to find the true position of a peak, measurements corresponding to midway between points of equal blackening on either slope of the record curve were found. If the curve is symmetrical the most probable value is given by the bisectors at different heights, as in table 1. Where the summits are slightly

* A. J. Bradley and A. H. Jay, *Proc. Phys. Soc.* **45**, 507 (1933).

† Obtained with a microphotometer made by the Cambridge Instrument Co., Ltd.

asymmetrical, figure 2*b*, the bisectors show a drift, table 2. Here the position of the peak is given as the value the bisector would have at the top of the curve. The examples mentioned above were the extreme cases of symmetry and asymmetry.

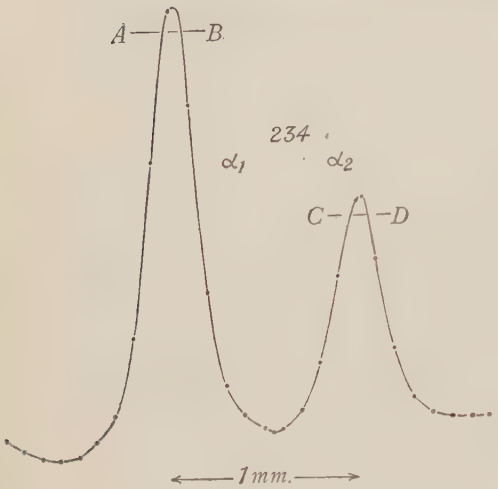

Figure 1*a*.

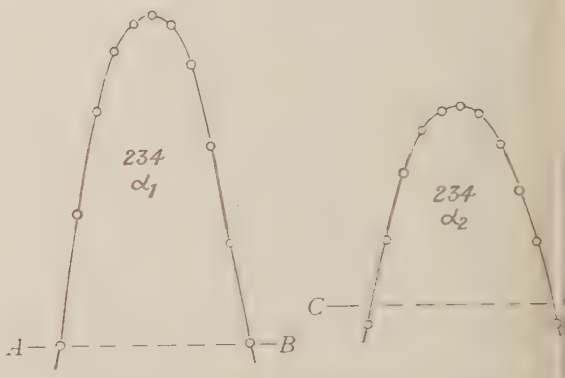

Figure 1*b*. Readings at intervals of 0·001 cm.

Figure 2*a*. Figure 2*b*.

Readings at intervals of 0·001 cm.

Table 1

Line 2, 3, 4α_1	Side 1	First adjustment
Position of points of equal blackening on either side of line. (cm.)		Bisector. (cm.)
0·9144	0·9226	½ (1·8370)
0·9150	0·9220	½ (1·8370)
0·9158	0·9211	½ (1·8369)
0·9170	0·9201	½ (1·8371)
	Peak: ½ (1·8370) = 0·9185 ± 0·0001 cm.	

Table 2

Line 2, 3, $4\alpha_2$	Side 1	Second adjustment
Position of points of equal blackening on either side of line (cm.)		Bisector. (cm.)
0·6171 0·6176 0·6184 0·6195	0·6258 0·6251 0·6241 0·6229	$\frac{1}{2}$ (1·2429) $\frac{1}{2}$ (1·2427) $\frac{1}{2}$ (1·2425) $\frac{1}{2}$ (1·2424)
	Peak: $\frac{1}{2}$ (1·2424) = 0·6212 ± 0·0002 cm.	

The film was reset in the photometer and a second series of measurements was made. The results for the 2, 3, $4\alpha_1\alpha_2$ doublet, for both sides of the film, are given in table 3.

Table 3

Side 1			Side 2		
Position of peaks (cm.) 2, 3, $4\alpha_2$	2, 3, $4\alpha_1$	Separation (cm.)	Position of peaks (cm.) 2, 3, $4\alpha_2$	2, 3, $4\alpha_1$	Separation (cm.)
First adjustment					
0·8186 ± 0·0001	0·9185 ± 0·0001	0·0999 ± 0·0002	0·3702 ± 0·0002	0·4700 ± 0·0001	0·0998 ± 0·0003
Sum of separations on both sides of film: 0·1997 cm.					
Second adjustment					
0·6212 ± 0·0002	0·7210 ± 0·0001	0·0998 ± 0·0003	0·6751 ± 0·0002	0·7748 ± 0·0001	0·0997 ± 0·0003
Sum of separations on both sides of film: 0·1995 cm.					
Average for two series of measurements: 0·1996 cm.					

The measurements in table 3 give the difference $S_2 - S_1$ between the S values for the 2, 3, $4\alpha_2$ and 2, 3, $4\alpha_1$ lines, S being the distance between corresponding lines on either side of the film. Before calculating the difference between the wave-lengths, it is necessary to correct the measurements $S_2 - S_1$ for a number of systematic errors. These arise from (i) the eccentricity of the specimen; (ii) the absorption of the radiation in the specimen; and (iii) the divergence of the X-ray beam. S_2, S_1 S

§ 3. CORRECTIONS FOR ECCENTRICITY OF SPECIMEN, ABSORPTION AND DIVERGENCE OF THE X-RAY BEAM

The glancing angle θ is related to the measurement S by the relation $S = 4R\theta$, R being the radius of the camera. It has, however, been shown that owing to film-shrinkage R cannot be considered to be a constant. It is most convenient to use for deriving θ from S a formula which allows for film-shrinkage. This formula* is $S/S_k = \theta/\theta_k$ where S_k is the length of film corresponding to a known angle θ_k, which θ, S, R S_k, θ_k

* A. J. Bradley and A. H. Jay, *Proc. Phys. Soc.* **44**, 563 (1932).

is a constant for the camera. The value of S_k depends in the same way as S on the film shrinkage.

The following measurements* were obtained by means of a travelling microscope:

$$S_k = 26{\cdot}443 \text{ cm. where } \theta_k = 82{\cdot}77^\circ.$$

$$S_1 = 24{\cdot}517 \text{ cm.}$$

The angle θ_1 corresponding to S_1 is given by the relation

$$\theta_1 = \theta_k \cdot \frac{S_1}{S_k}$$

$$= 76{\cdot}741^\circ.$$

The value of θ for the 2, 3, $4\alpha_1$ reflection can also be calculated from Bergqvist's† value of the d_{100} spacing of quartz, and the axial ratio $c/a = 1{\cdot}10002$*. The calculated

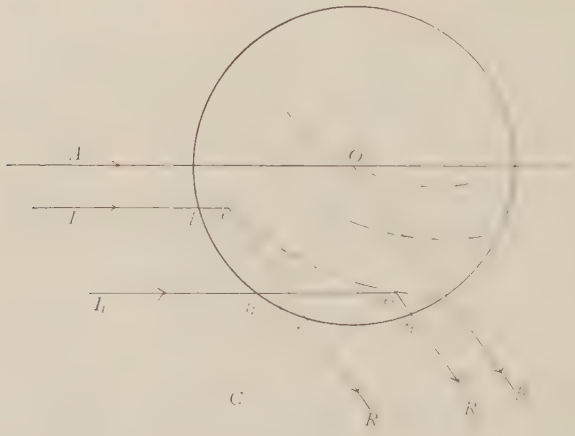

Figure 3.

value of θ after correction for refractivity is 76·790°. The difference 0·049° between the observed and calculated values is due to the systematic errors, and may be used as a measure of the corrections which must be applied to the doublet separation.

The systematic errors arise chiefly from the eccentricity of the specimen, but a small additional effect, introduced by absorption, may be calculated from its known radius.

A simple examination, following the method of Claasen‡, shows the effect of absorption on the line-position. Reference is made to figure 3. A powdered specimen of uniform density§ is bathed in a parallel beam of X-rays. AO is the direction of the incident beam and OB the direction of the reflected beam. Consider the incident beam I and the reflected beam R. The beam I enters the specimen at d, travels a distance de before reflection and a distance ef after reflection, and leaves the

* A. J. Bradley and A. H. Jay, *Proc. Phys. Soc.* **45**, 507 (1933).
† O. Bergqvist, *Z. f. Phys.* **66**, 494 (1930).
‡ A. Claasen, *Phil. Mag.* **9**, 57 (1930).
§ Continual rotation of the specimen simulates this condition.

specimen at f. The distance travelled through the specimen is $(de + ef)$. From any point of scattering e, the intensity of the reflected beam will be given by

$$I = kI_0 e^{-\mu(de+ef)}.$$

Similarly for the ray $I_1 d_1 e_1 + e_1 f_1 R_1$

$$I = kI_0 e^{-\mu(d_1 e_1 + e_1 f_1)}.$$

Other factors being equal, the intensities of the reflected beams are equal when $(de + ef) = (d_1 e_1 + e_1 f_1)$.

Scattering points which contribute equal effects lie on a curved surface, such that the sum of the distances travelled through the specimen before and after reflection is a constant. There are an infinite number of these surfaces, each having its own path-difference. The curves are symmetrical about a direction OC, where $A\hat{O}C = C\hat{O}B$.

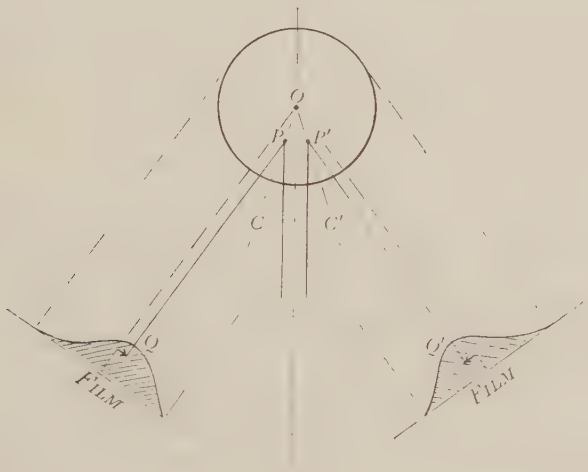

Figure 4.

Consider a line on the film, the peak of which has been displaced to a higher angle owing to absorption of the radiation in the specimen, see figure 4. The peak may be regarded as coming from the point P, the centre of the scattering source. P will be displaced from the centre of the specimen in the direction OC. Similarly, for the corresponding reflection on the other side of the film the centre of scattering may be regarded as being at some point P^1.

It is difficult to make an accurate estimate of the shift OP, which must depend on the size of the specimen, absorption of the radiation in the specimen, and the position of the line on the film.

The use of a thin diluted specimen of transparent material, such as quartz, with copper K_α radiation, reduces this shift to a small amount, and the high-angle reflections will tend to be symmetrical in shape. In fact photometer measurements show that the quartz line $h, k, l = 2, 3, 4$ is symmetrical to a high degree. The displacement OP must therefore be small.

Since the two scattering centres P and P^1 are not on the axis of the system, an error is introduced by the divergence of the X-ray beam from the point X, figure 5. The ray PQ for a parallel beam is swung round to PS, making QPS equal to ϕ. The angle ϕ is given by $\sin \phi = OP.\sin P\hat{O}X$ PX. Consider a specimen, 0·3 to 0·4 mm. in diameter, giving a reflection at $\theta = 75°$. Suppose that $PX = 10$ cm. and assume that OP is one quarter of the radius of specimen, then $\phi = 0{\cdot}006°$. The measured angle of deviation is then too large by a ϕ, and so the measured glancing angle is too large by an amount 0·003°. The angle would therefore be 76·738° for a parallel beam.

A small correction in the separation of the doublet is also necessary because of the displacement of the two scattering centres P and P^1, one for either side of the film, from the centre O of the specimen. The displacement OP or OP^1 may be resolved into two components, (i) a displacement OO^1 along the axis of the system,

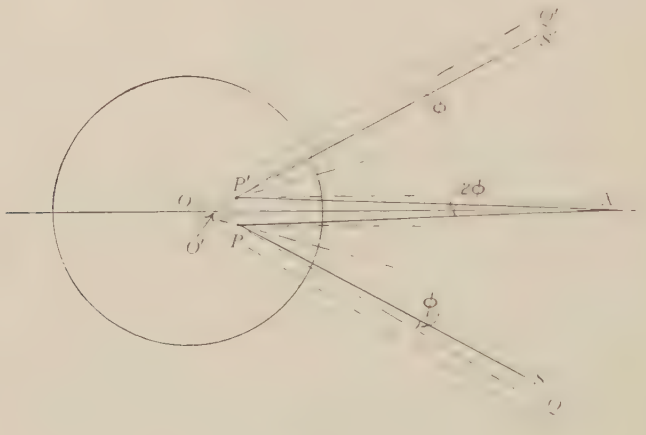

Figure 5.

which simply introduces a numerical modification in the eccentricity factor, and (ii) a displacement O^1P or O^1P^1. The second effect is considered separately. Its effect is to increase the doublet separation in the ratio $(R - O^1P) : R$. The correction to be applied in the present instance is 0·0001 cm., giving a doublet separation of 0·1997 cm.

Eccentricity of scattering source. The shift of the lines on the film to smaller measured angles, for instance the shift of 2, 3, 4α, from 76·790° to 76·738°, is due to the displacement of the scattering source away from the incident beam. In figure 6 a line is displaced a distance x in the direction of the arrow, owing to a shift in the position of the specimen. The distance x may be calculated from the error (76·790°–76·738°) in the measured angle for 2, 3, 4α, and the radius 4·576 cm. of the film. Thus

$$x = R.\sin 2\,(76{\cdot}790°–76{\cdot}738°).$$

dR This shift is equivalent to an increase dR in the effective radius of the film at this point, where

$$x = dR.\tan(\pi - 2\theta).$$

Combining the above equations, we find $dR = 0{\cdot}017$ cm. The observed separation of the component lines of the doublet is proportional to the distance of that part of the film from the scattering source, and is therefore increased in the ratio dR/R. Hence the observed separation 0·1997 cm. is 0·0007 cm. too great, and the correct value is 0·1990 cm., equivalent to an angular separation $(\theta_2 - \theta_1)$ of 0·6229°.

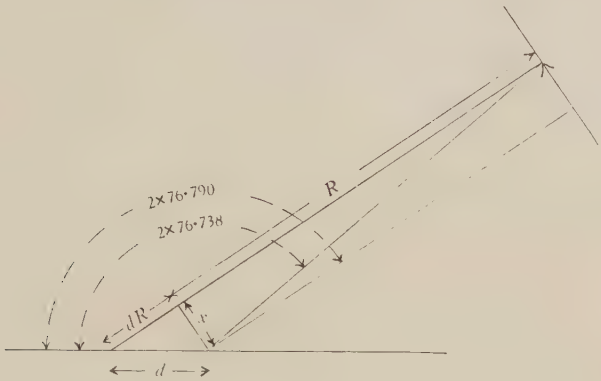

Figure 6.

§ 4. CALCULATION OF $\lambda_2 - \lambda_1$

From the Bragg relationship $2d.\sin\theta = n\lambda$ we have the difference in wave-lengths given by the expression

$$\lambda_2 - \lambda_1 = \lambda_1 \frac{\sin\theta_2 - \sin\theta_1}{\sin\theta_1}$$

$$= \frac{2\lambda_1}{\sin\theta_1}.\cos\left(\frac{\theta_2 + \theta_1}{2}\right).\sin\left(\frac{\theta_2 - \theta_1}{2}\right).$$

The accuracy of $(\lambda_2 - \lambda_1)$ is governed almost entirely by the angular separation $(\theta_2 - \theta_1)$, which is equal to 0·6229°. By substitution in the above equation the value of $\lambda_2 - \lambda_1$ is given as 3·833 X. for copper $K_{\alpha_1\alpha_2}$ radiation.

The data in table 4 for copper $K_{\alpha_1\alpha_2}$ radiation are taken from the latest edition of Siegbahn's *Spektroskopie der Röntgenstrahlen**.

Table 4

Year	Wave-lengths (X.)		Difference (X.)	Author
	λ_2	λ_1		
1924	1541·19	1537·32	3·87	Lang
1925	1541·02	1537·29	3·73	Leide
1926	1541·15	1537·30	3·85	Schror
1929	1541·243	1537·396	3·847	Siegbahn
1930	1541·232	1537·395	3·837	Wermerlof

The value now obtained, 3·833 X., is in very good agreement with the latest of the above values.

* Julius Springer, Berlin (1931).

§ 5. ACKNOWLEDGMENT

I wish to thank Prof. W. L. Bragg, F.R.S. and Dr A. J. Bradley for their encouraging interest in the work, which was carried out in the Physical Laboratories of the University of Manchester. My thanks are also due to Mr A. P. M. Fleming, C.B.E., Director-Manager, Research and Education Departments, Metropolitan-Vickers Elec. Co., Ltd., and to Dr T. Swinden, Director of Research, United Steel Co., Ltd., with whom I am now associated.

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A TIME-BASE CIRCUIT AND ELECTRON RELAY FOR USE WITH A CATHODE-RAY OSCILLOGRAPH*

By A. MORRIS CASSIE

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ABSTRACT. The ordinary sealed-off type of cathode ray oscillograph is adapted to form a relay capable of picking out accurately points on a current or voltage wave to be investigated by means of a second cathode-ray oscillograph. A time-base circuit and the amplifier for the relay are described also.

§ 1. PRINCIPLES OF THE METHOD

EARLIER work of the British Electrical and Allied Industries Research Association on the shape of the voltage-recovery wave at arc-extinction, was based on experiments made with a circuit of very small scale, an ordinary sealed-off cathode-ray oscillograph, and repeating phenomena. Methods employing an oscillograph capable of giving transient records of single voltage-rises had therefore to be developed. This note is concerned with the technical problems which have arisen in connexion with these experiments and it is hoped that their solution may have applications in other electrical and particularly in switch-gear problems.

On the Wood oscillograph employed the recording space is very limited, being that of a photographic plate $2\frac{1}{8}$ in. by $1\frac{5}{8}$ in. To get a record of recovery voltage lasting about 2×10^{-4} sec. the spot must traverse the plate in, say, 5×10^{-4} sec. This is, of course, a very moderate speed of recording for the present-day type of cathode-ray oscillograph. The difficulty that arises is that the spot cannot be allowed to oscillate backwards and forwards along the plate, waiting for the phenomenon to happen, as this would cause excessive fogging of the plate. From twenty to thirty traverses would be made for every half-cycle at standard frequency. A further drawback of this procedure is that the time scale would almost necessarily have to be sinusoidal, and therefore inconvenient for measurement; moreover it would render almost useless any record which happened to come at either end of the scale where the speed of traverse is small. On the other hand the type of phenomenon investigated by means of the high-voltage cathode-ray oscillograph invariably takes the form of a very rapidly rising voltage of high value. This allows of the use of the three-spark-balls method of tripping the time-scale mechanism. Generally a condenser is charged up to a high voltage and has in parallel with it the time deflection plates of the oscillograph, together with a resistance in series with the two outer of the three spark balls. The centre ball is connected in some way to the circuit

* Report G/XT43 of the British Electrical and Allied Industries Research Association.

in which the high-voltage wave to be investigated occurs. When the surge arrives the rapid change of potential of the centre ball upsets the equilibrium, so that a spark takes place across the two gaps in succession, the capacity is discharged and the time-scale sweep is thus operated. In the mean time the phenomenon passes along a delay cable so as to arrive at the other pair of deflection plates of the oscillograph just after the time-scale sweep has been started.

In the present investigation this scheme had to be ruled out for various causes. For instance, the voltage-changes are not necessarily characterized by a steep wave-front, and though there is such a front at arc-break the "arcing volts" previous to it must also be recorded. For this purpose a long delay cable would be necessary, and this would alter the nature of the voltage investigated.

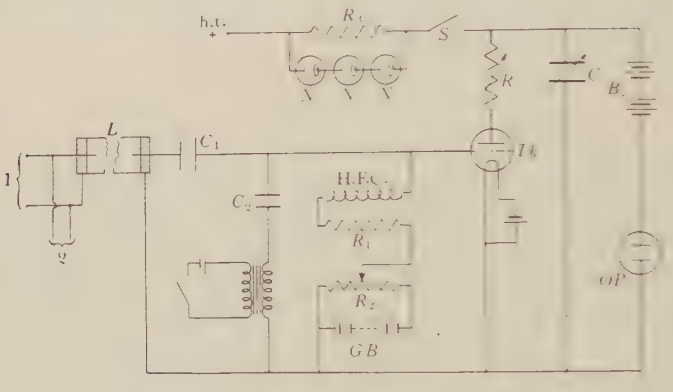

Figure 1. Thyratron time-scale circuit.

The time-scale circuit actually used is in principle the same as that described above. The spark balls are replaced by a thyratron relay, and the potential-change on its grid which causes it to trip at the required moment is not initiated by the voltage surge but by a special cathode-ray relay tube operating in conjunction with the current in the switch circuit which is a locked quantity inasmuch as the voltage surges and oscillations to be investigated occur just before and after a current zero.

Figure 1 shows the time-scale circuit in detail. The condenser C is charged up to about twice the potential required to deflect the spot completely across the recording plate of the oscillograph. The deflecting plates are at OP and a bias battery B_2 is used to adjust the initial position of the spot, which is just off the recording plate. The potential on C is maintained through the resistance R_3, which is several megohms, the switch S being closed. By means of the potentiometer R_2 and grid-bias battery GB the grid of the thyratron Th is kept at a potential with respect to its filament which is 5 or 10 volts more negative than the tripping potential. The tripping impulse, which must be positive, is supplied by an amplifier at terminals 1 and passes along a shielded low-capacity line L to a condenser C_1 of 0·01μF. and then on to the grid. A high-frequency choke H.F.C. and a grid-bias resistance R_1 are inserted between the grid and the potentiometer R_2 to prevent the impulse from passing back to earth.

On the arrival of the impulse on the grid the thyratron flashes, and the charge from the condenser C flows through the non-inductive resistance R. The potential across C therefore falls off exponentially, exactly the same fall occurring across the deflection plates OP. Normally the thyratron would be extinguished after this discharge since the current from the h.-t. batteries or rectifier through R_3 cannot be made large enough to keep the thyratron burning without affecting the exponential discharge of C adversely. R_3 is, however, so large that the recharging of condenser C takes place more rapidly than the closing of the shutter of the camera, with the result that bad over-exposure and fogging of the plate occur. To avoid this several neon lamps N are placed in parallel with R_3. When the potential on C has fallen to a value at which the spot has been swept completely off the plate, the neon lamps flash, passing enough current to keep the thyratron alight but not enough to cause an appreciable drop of potential across R. The strength of the current is adjusted to the desired value by the use of considerable resistance in series with the neon lamps themselves. In this way the spot does not return after the thyratron has been tripped. After the camera shutter has been closed, switch S is opened and closed and the time-scale is ready for the next test.

§ 2. THE TRIPPING-MECHANISM

The first method of tripping tried out was based on a suggestion made by Mr A. P. Paton of the Electrical Research Association. The beam of light from an electromagnetic oscillograph recording the current in the main circuit was allowed to fall on a photoelectric cell as the vibrating mirror passed through its zero position. To make the timing sufficiently accurate, the velocity of the beam at the cell had to be large and a very narrow stop in the form of a slit had to be placed in front of the cell. This caused the light-flux to be so small that the electric impulse given by the cell was too small to be amplified successfully.

The principle of the method as now used for tripping the time scale will be understood by reference to figure 2. The heavy lines represent the main circuit containing the switch S under investigation. CT is a current transformer which supplies current to two coils C_1 and C_2 placed on either side of a special relay tube. This is essentially a permanent vacuum cathode-ray oscillograph, but instead of having a large conical end with a fluorescent screen the tube is cylindrical, somewhat longer than usual and has an electrode E at its end. When alternating current flows in the main circuit and therefore in the coils C_1 and C_2 the spot, which in the undeflected position falls on E, is deflected perpendicularly to the plane of the paper passing across E each time the current passes through a current zero. This causes a charge to fall on E at each current zero. E is connected to the grid of the first valve of an amplifier A and the output of the amplifier is used to trip the thyratron and initiate the time-scale movement as explained above. To prevent this from happening before the occurrence of the current-zero at which the record is to be taken, the output of the amplifier is temporarily short-circuited. As the switch S opens, its moving contact opens a selector switch SS after it has travelled a selected

distance. *SS* then operates a low-capacity switch by means of a thyratron relay *SR*, opening the short-circuit on the amplifier. The voltage across the switch at the next current-zero is therefore recorded. With a gas-blast switch, as developed by the Electrical Research Association, there is no difficulty whatever in predetermining the current-zero at which the arc will be extinguished, so that records of the recovery voltage at the final current zero can be obtained. With the ordinary plain break in oil, however, nothing of the kind can be done. This method of recording then becomes a hit-or-miss method. For plain break in oil, therefore, a special resetting time scale which will not fog the photographic plate during resetting will have to be constructed*, and several voltage records must be made on the same photographic plate at each test.

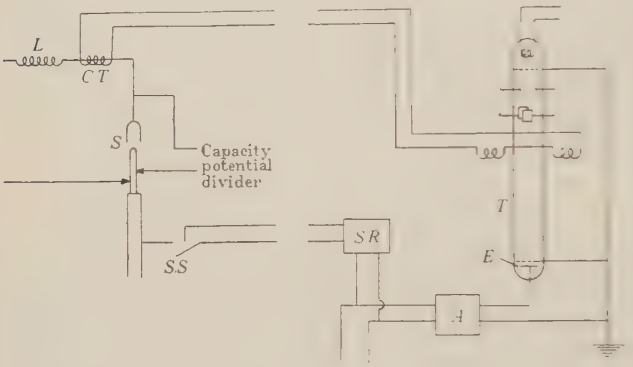

Figure 2. General scheme of tripping circuit.

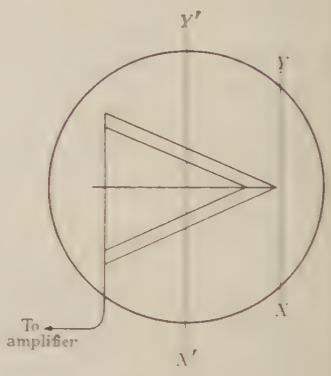

Figure 3.

The electrode E and the amplifier A are of special construction. Figure 3 represents the end of the tube on which in normal running the phenomenon is recorded, i.e., the fluorescent screen. The electron stream is, of course, perpendicular to the plane of the paper and represented by a spot on figure 3. The deflection coils C_1 and C_2, figure 2, are arranged on the oscillograph so that when current flows in them the spot is deflected along the line XY, figure 3. With a sinusoidal current flowing in the coils the spot therefore moves up and down XY in simple harmonic motion. With a frequency of 50 c. sec. the velocity of the spot as it passes through its zero position (corresponding to the zero of current) is $314A$ cm. sec., where A is the amplitude of the swing in centimetres. A can be altered at will by changing the number of ampere-turns in the coils. The time taken by the spot to move 1 mm. is thus $0{\cdot}1\ 314A$ sec., so that the amplitude required to make it move 1 mm. in 10 microseconds is 32 cm. approximately. There is no difficulty in producing such an amplitude. Even if the tolerance in the tripping-time were 1 μsec., requiring an amplitude of 320 cm., the apparatus would function perfectly provided harmonics in the current wave were not too great. Corrections could be made by altering the phase of the current in the coils. When the correction, as is generally

* This has now been done, 7/8/1934.

the case, would mean an advancement of the current in the coils, a very simple arrangement is used to effect it.

On reference again to figure 3 it will be seen that the electrode has a special shape. AB and AC are two strips of metal foil, AO and BC metal wires. AO is perpendicular to the line XY along which the deflections of the spot take place. The spot is set somewhere along AO, say at M, which is then its zero position. It moves along $X'\,Y'$ when current flows in the coils. As the spot approaches from the upper half of the half-cycle, it meets the electrode at b before it has come to its zero position M. The impulse which trips the time-scale therefore occurs $L/3142A$ sec. before the actual current-zero, where L is the distance Mb in millimetres. The same time-advancement is obtained when the current approaches zero from the opposite direction. To vary L all that need be done is to shift the zero position of the spot along AO, a very simple operation. Hence the time scale can be tripped at any preselected time before current-zero*.

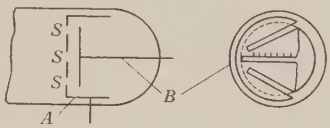

Figure 4.

This type of electrode, however, had a serious defect. In a sealed-off tube, where a certain amount of gas focusing is relied upon, positive-ion space charges surround the electron stream itself. The tube also depends for its successful operation on secondary emission from the fluorescent screen forming the return path to the anode. Hence these additional electrodes, being practically at earth potential which is the potential of the anode, act as collectors for all such charges formed in the neighbourhood. The result is that no sudden change of potential takes place when the electron stream proper falls on such an electrode. The potential rises gradually as the beam approaches it and this rise in potential is much too slow to be of any use in the accurate timing required. The electrode system was therefore modified as shown in figure 4.

A similar method of producing a time-advancement is incorporated, but three slots in an earthed shield A now take the place of the electrode in figure 3. The earthed shield collects practically all the stray current and all the current from the beam except when the electrons fall on one of the slits, when they pass through to the electrode B, which is connected to the amplifier. A scale for positioning the spot is provided along one edge of the centre slot. The divisions are 5 mm. apart†.

The electrode of the cathode-ray relay tube cannot be connected directly to the grid of the thyratron, as the impulse is in the wrong direction. A single-valve amplifier may be interposed but much better results can be obtained by using a three-valve amplifier‡ of special design.

* See Appendix 2.

† These tubes were made by Mr L. H. Bedford of A. C. Cossor and Son.

‡ Appendix 3.

Alternating current of 50 c. sec. is passed through the deflecting-coils of the cathode-ray relay tube and also through a similar pair of deflecting-coils on another cathode-ray oscillograph of the sealed-off type. In the relay tube the spot is thus caused to move simply-harmonically along a line perpendicular to the centre slot in the earthed shield. The electric impulses falling on the shielded electrode as the beam sweeps across are amplified and applied to one of the deflecting plates of the oscillograph so as to produce deflections perpendicular to that produced by its

Figure 5.

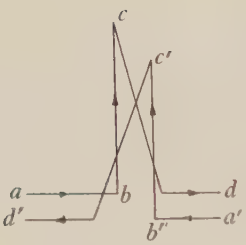

Figure 6.

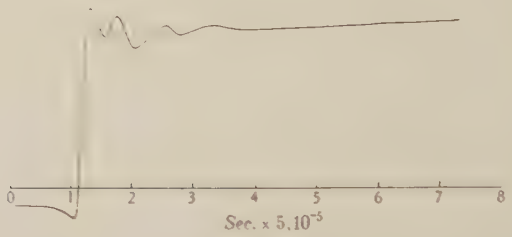

Figure 7. Trace of cathode ray oscillogram of voltage across h.-t. switch at current-zero, the time-scale device of oscillograph being tripped by the electron relay tube.

current coils. In this way a stationary picture showing the impulses as the spot traverses the slots in the relay is obtained. In figure 5 six prominent peaks are seen corresponding to the six edges of the three slots*. Those shown correspond to potentials of about 50 V. from the zero line.

The accuracy with which the thyratron can be tripped depends on the steepness with which the voltage rises to one of these peak values. This can be determined by comparing the two peaks corresponding to any one of the three slots. In figure 6, *ab*

* What appears to be a seventh prominent peak as well as other raggedness in the picture is due to a complicated mixture of interference and under-damping of the amplifier. This has no effect on the operation of the apparatus.

represents the beam approaching the left-hand edge of a slit, *bc* represents the impulsive rise when the beam crosses the edge and *cd* the dying away of the impulse. When the beam approaches the slit from the opposite direction the sequence of events will be represented by $a'b'c'd'$, where bb' will be the width of the slit. Now if the rises were instantaneous *bc* and $b'c'$ would be parallel, but if they were slow cc' would be less than bb'. Hence by measuring these quantities the time taken by the voltage to rise from *b* to *c* can be deduced.

Figure 7 is an oscillogram taken by this method. The timing-wave has a frequency of 2×10^4.

§ 3. ACKNOWLEDGMENTS

The author is greatly indebted to Mr L. H. Bedford of A. C. Cossor and Son, who was responsible for the making of the relay tube. Thanks are also due to Mr L. Gosland, of the Electrical Research Association, whose help was invaluable in solving the many problems of electric and magnetic interference which arose.

APPENDIX 1

DISCHARGE OF CONDENSER THROUGH THYRATRON

The thyratron constitutes a back e.m.f. of about 16 V. during the discharge of the condenser. If q is the charge on the condenser at any instant and ϵ the back e.m.f. of the thyratron then

q, ϵ

$$R\frac{dq}{db} + \epsilon + \frac{q}{C} = 0,$$

$$q + \epsilon C = Ae^{-t/RC},$$

$$e = Ee^{-t/RC} - \epsilon\,(1 - e^{-t/RC}),$$

where e is the e.m.f. across the condenser at any instant and E the e.m.f. of the h.-t. battery. Since E is about 700 V. and $\epsilon \doteqdot 16$ V. the effect of the thyratron becomes important only near the end of the time scale.

APPENDIX 2

ADVANCEMENT PRODUCED BY THE PHASE-ADVANCER

We assume firstly that the main circuit current is sinusoidal, that no third harmonic is present, and that the current transformer functions perfectly. As the anode potential can be varied usefully in making the settings, the sensitivity may conveniently be defined by the equation

$$\sqrt{500}\, S_{cm}/V_c^{\frac{1}{2}}$$

where S_{cm} is the deflection in centimetres for 1 ampere-turn at an anode potential of 500 V., and V_c is the anode potential applied to the relay tube.

V_c

i A If i is the r.m.s. current in the deflecting coils and n the number of turns, then the amplitude A of swing of the spot will be given in centimetres by

$$A = \sqrt{2}.ni\sqrt{500}.S_{cm}V_c^{-\frac{1}{2}}.$$

v The velocity v of the spot near its zero position is therefore given by

$$v = 314\sqrt{2}.\sqrt{500}.ni\,S_{cm}/V_c^{\frac{1}{2}}.$$

d T If d is the distance of the outer edge of the slot from the centre line the advancement-time T will be given in seconds by

$$T = V_c^{\frac{1}{2}}\,d/314\sqrt{1000}.ni\,S_{cm}.$$

x K Using in place of d the more convenient quantity x shown in figure 8, and writing

$$d = Kx,$$

we have

$$T = Kx\,V_c^{\frac{1}{2}}/314\sqrt{1000}.ni\,S_{cm}.$$

This time T is the time required to bring the indication at current-zero to a convenient position on the photographic plate. The time-scale sweep starts at A, figure 9, just off the recording-plate, and ends at B, which is well off the plate, so that the sweep over the plate is roughly linear.

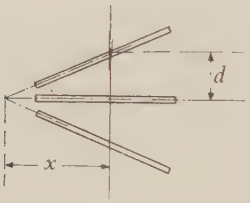

Figure 8.

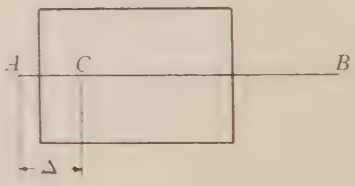

Figure 9.

As the anode-potential of the main oscillograph may be varied also, its electrostatic sensitivity can be conveniently expressed in cm. per volt by the equation

$$5 \times 10^3\,S_s/V,$$

S_s where S_s is the deflection in centimetres for 1 V. on the deflecting-plates when the anode voltage is 5,000.

l Δ Hence if l is the total length of the sweep and E the e.m.f. of the h.-t. battery used in producing the sweep*, the distance Δ through which the spot has moved from A in time T will be given by

$$\Delta = l - le^{-T/RC} + \frac{l\epsilon}{E}(1 - e^{-T/RC}).$$

To a first approximation, when T/RC is small compared with unity, the effect of the thyratron being left out of account,

$$\Delta = lT/RC$$
$$= \frac{5 \cdot 10^3 S_s}{l} E\,\frac{T}{RC}.$$

* See Appendix 1.

This assumes that there is no time lag between the instant at which the spot strikes the edge of the slot and the tripping of the thyratron; but the lag will become appreciable in very fast time scales and it will then be necessary to write $T - \tau$ for T in the above equation, thus

$$T = \tau + \Delta RCV/5 \cdot 10^3 S_s E.$$

Hence

$$\frac{Kx V_c^{\frac{1}{2}}}{314 \sqrt{1000} \cdot ni\, S_{cm}} = \tau + \frac{\Delta RCV}{5 \cdot 10^3 S_s E}.$$

All these quantities can be measured experimentally except τ. A group of values of x, n, i and Δ or x and Δ only, all other quantities being kept fixed, will enable a graph to be plotted giving the value of τ. In the experiments made with our equipment the presence of the third harmonic in i and of imperfection of the current transformer caused systematic errors greater than τ. These, however, can be allowed for by graphing groups of results in the same manner as for τ.

For experimental work where i varies from test to test it has been found convenient to write

$$x = \zeta\, RCVni/V_c^{\frac{1}{2}}$$

where ζ varies with the current in the main circuit. The deflecting coils are easily changed for others having a different number of turns, or may be connected in series or parallel. ni is chosen so as to make $\zeta\, RCVni/V_c^{\frac{1}{2}}$ as nearly equal to the mean value of x as possible, and the final adjustment is made with x.

APPENDIX 3

THE VALVE AMPLIFIER

The design of the amplifier, shown diagrammatically in figure 10, is very important from the point of view of reproducing the very steep peaks as accurately as possible. In fact the amplifier can be made to sharpen them up and it is on this

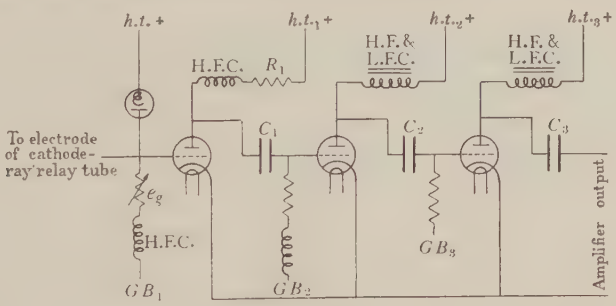

Figure 10.

account only that the relay tube works as successfully as it does. Two points must receive special attention in design in order to produce the desired results. Firstly, a steady potential applied to the grid of the first valve must not be reproduced as a steady potential across the output, while any changing potential on the first grid must be reproduced as a potential-maximum or minimum. The second point is that the potential on the first grid may mount up to values dangerous for the valve, as the

electrode of the relay tube can rise or fall to the anode-potential of the tube, generally 500 to 800 V. As the d.-c. amplification is not quite negligible, this growth of grid-potential may have other results besides endangering the valve. Hence a grid leak of small resistance, or the neon-lamp device described later, must be used.

The grid leak consists of a variable low-capacity resistance of about 10,000 Ω. A low-resistance variable high-frequency choke would do even better service here. One electrode of a decapped neon-lamp is also connected to the grid, the other electrode being connected to about 140 V. h.-t. positive. Hence if the grid becomes more than 30 V. or so negative with respect to the filament of the valve, the neon strikes, thereby protecting the grid from further increase of negative voltage. This only happens when the beam of the cathode-ray tripping-tube is kept directly on the shielded electrode. Generally the low resistance of the grid leak is sufficient protection; its effect is to allow the grid to return rapidly to equilibrium potential after receiving an impulse, such impulses being all of the same sign.

The anode circuit of the first stage consists of a resistance and high-frequency choke with capacity coupling to the grid of the next valve. The anode circuits of succeeding stages are choke-capacity-coupled. This gives the desired emphasis to de/dt rather than to e.

When the potential of the grid becomes suddenly more positive the anode current increases. At first this increase in current will cause a fall of potential on the grid of the next valve, the impedance of the choke to changes of current being high. When steady conditions return, the potential at the anode returns to the battery-potential if the d.-c. resistance of the choke is negligible, and so the potential on the second grid returns to its original value also, even though the potential on the first grid remains changed.

i_p, e_g, e_p

Values of circuit constants. In ordinary valve-circuit notation the increase in plate current i_p due to a change of grid-potential e_g and of plate-potential e_p is given by

$$i_p = G_{cp} e_g + G_p e_p,$$

G_{cp}, G_p

where G_{cp} is the mutual conductance and G_p the plate conductance of the valve. i_p is made up of two parts, one from the plate-circuit choke and one from the grid capacity of the next valve; hence

$$i_p = i_c - i_x;$$

$$e_p = -L \frac{di_c}{dt};$$

and

$$i_x = -C \frac{de_p}{dt},$$

where C is the grid capacity of the next valve.

From these equations we have

$$\frac{di_p}{dt} = \frac{di_c}{dt} + \frac{di_x}{dt}.$$

$$\therefore G_{cp} \frac{de_g}{dt} + G_p \frac{de_p}{dt} = -\frac{e_p}{L} - C \frac{d^2 e_p}{dt^2}.$$

$$\therefore C \frac{d^2 e_p}{dt^2} + G_p \frac{de_p}{dt} + \frac{e_p}{L} = -G_{cp} \frac{de_g}{dt},$$

or, in operational notation,

$$(D - m_1)(D - m_2)\, e_p = -\frac{G_{cp}}{C} D e_g,$$

where

$$m_1, m_2 = -\frac{1}{2C}\left\{G_p \pm \sqrt{\left(G_p^2 - \frac{4C}{L}\right)}\right\}.$$

The best case will clearly be when e_g is a unit function of the time. Then

$$e_p = -\frac{G_{cp} e_g}{C(m_1 - m_2)}\left[\frac{D}{D - m_1} - \frac{D}{D - m_2}\right] 1$$

$$= -\frac{G_{cp} e_g}{C(m_1 - m_2)}\left[\epsilon^{m_1 t} - \epsilon^{m_2 t}\right].$$

Write

$$m_1 = -\alpha + \beta \quad m_2 = -\alpha - \beta$$

so that

$$\alpha = G_p/2C, \quad \beta = \sqrt{(G_p^2/4C^2 - 1/LC)},$$

$$\therefore\ e_p = -\frac{G_{cp} e_g}{C\sqrt{(G_p^2/C^2 - 4/LC)}}\, e^{-\alpha t}\left(e^{\beta t} - e^{-\beta t}\right) \qquad \ldots\ldots(1).$$

There are two ways in which this expression for e_p may be made to approximate initially to a unit function, i.e. in which $(de_p/dt)_o$ may be made as large as possible. Physically these two methods are (1) by making L very large and C very small, and having valves of as high mutual conductance G_{cp} as possible. It is clear that any increase in the plate current will cause charge to flow from the condenser C rather than through the inductance L, thus rapidly altering the potential on the next grid. G_p must be chosen so that the circuit is not oscillatory. (2) By making the circuit highly oscillatory, i.e. with a very high natural frequency, L and C being both small. Changes in i_p amount to the application of impulses to this circuit, causing it to oscillate at this high frequency. This arrangement too will give a high rate of change of $(de_p/dt)_o$. There is a possibility, however, that the amplitude of this oscillation may be so small as to give less satisfactory results than the previously described arrangement. The former arrangement is indeed the better; this can be shown as follows. From equation (1) it is clear that the condition to be satisfied is that β must be real and numerically less than α, and further the difference between α and β must be as small as possible, i.e. $4/L$ must be small in comparison with G_p^2/C. Hence G_p and L must be large and C small. Since this is to be the case we may write

$$\beta = (\alpha^2 - K^2)^{\frac{1}{2}} = \alpha(1 - K^2/\alpha^2)^{\frac{1}{2}}, \quad K = 1/\sqrt{(LC)}$$

$$= \alpha(1 - K^2/2\alpha^2) = \alpha - \delta$$

where

$$\delta = K^2/2\alpha^2 = 1/2LC\alpha^2.$$

Then

$$\alpha + \beta \fallingdotseq 2\alpha \text{ and } \alpha - \beta = \delta,$$

and equation (1) can be written

$$e_p = -\frac{G_{cp}\, e_g}{C\sqrt{(G_p^2/C^2 - 4/LC)}}\left(\epsilon^{-\delta t} - \epsilon^{-2\alpha t}\right) \qquad \ldots\ldots(2).$$

For a rapid rise in e_p, therefore, α must be as large as possible. If most of the rise is to occur in time t_e then $2\alpha t_o = 1$, and if $t_o = 10^{-6}$ sec. then $2\alpha = G_p/C = 10^6$. t_e If $G_p = 10^{-4}$ mho., C would have to be 10^{-10} F., which is possible, C being the grid earth capacity of the valve.

The amplitude factor in equation (2) contains G_p in the denominator and it therefore decreases as G_p is increased, so that G_p must be made small and $G_p C$ large. Hence the principal practical requirement in the construction of the amplifier is to have C as small as possible, and then to choose G_p so as to make G_p equal to $10^6 C$. Next $4/L$ must be very much less than $G_p^2 C$; for instance, $L = 400 C G_p^2$, while $C = 10^{-11}$ F., $G_p = 10^{-5}$ mho.; and L would have to be 40 henries. These are all practicable values.

The oscillatory arrangement will be considered briefly. In equation (1) write

$$\beta = \frac{1}{2C}\sqrt{\left(\frac{4C}{L} - G_p^2\right)}.$$

Then

$$e_p = -\frac{G_{cp} e_g}{j\beta} \epsilon^{-\alpha t} [\epsilon^{j\beta t} - \epsilon^{-j\beta t}]$$

$$= -G_{cp} e_g \sqrt{(LC)} . e^{-\alpha t} \sin \beta t.$$

The oscillation frequency must be of the order of 10^6 c. sec., i.e. β must be of the order of $5 . 10^6$. Since β comes in the denominator of the amplitude term, the latter will be extremely small, about 10^{-11} of the value of the same term in the other arrangement.

DISCUSSION

Prof. G. I. Finch said that the author's device might be usefully adapted to the control of switch-gear for the purpose of opening a circuit at the instant when the current is passing through its zero value.

Mr R. S. Whipple said that the method would save a good deal of expense by reducing the amount of film that normally had to be expended to ensure that a particular phenomenon of brief duration was recorded.

REVIEWS OF BOOKS

Introduction to Theoretical Physics, by J. C. SLATER and N. H. FRANK. Pp. xx + 576. (New York and London: McGraw-Hill Publishing Co., Ltd., 1933.) 30*s*. net.

This is an excellent book, particularly well designed to supplement existing English texts, from which it differs radically in treatment. It might well, as the authors point out in their preface, have been alternatively entitled "An introduction to the methods of theoretical physics," and it may be especially recommended to those students who already have an adequate knowledge of experimental physics and who wish to become more closely acquainted with the technique of the mathematical side of their subject.

More than half of the book is rightly devoted to a concise but reasonably consecutive account of the classical mechanics and theoretical physics. This is treated on modern lines, and leads naturally to the development and applications of wave and statistical mechanics to which chapters 28–42 of the book are devoted. Within the obvious limitations of space the treatment is remarkably comprehensive; and the value of the book to students is enhanced, and the text to some extent supplemented, by an extensive collection of exercises and problems.

Introduction to Modern Physics, by F. K. RICHTMYER. Pp. xviii + 747. (London and New York: McGraw-Hill Publishing Co., Ltd.) 30*s*. net.

The author of this, one of the most important of modern text-books, has earned the thanks of the present generation of learners and teachers, in that he has refused to alter seriously the scope of the book. As users of the work are well aware, the first three chapters, some eighty pages in all, are concerned with an admirable historical treatment which leads us from Thales to Maxwell. The rush of modern discovery must have sorely tempted the author to cut down his introduction in order to admit the work of yesterday, but he has wisely resisted the temptation, even at the price, for example, of the omission of any discussion of cosmic radiation. The work develops on orthodox lines. Following the historical introduction, we have chapters on electromagnetic theory, the photoelectric effect, the quantum theory and its application to specific heats, spectra and the nuclear atom, the vector model of the atom, X-rays, the nucleus, and a final chapter on wave-theory. In this edition the chapter on the photoelectric effect has been extended and that on the nucleus re-written. The chapters on the wave-theory and on the vector model of the atom are new.

It will be seen that the author has interpreted the term "modern" in no narrow sense, and his scholarly, *and thoroughly physical*, treatise is to be welcomed as a valuable addition to our pedagogical literature. A. F.

L'Univers en Expansion, by Sir ARTHUR EDDINGTON, translated by J. ROSSIGNOL. Pp. xii + 166. (Paris: Hermann et Cie., 1934.) 15 francs.

This is the French translation of that very famous work, *The Expanding Universe*. Mainly intended for the intelligent layman, it has also a particular interest for the physicist. If any there be who, like the present reviewer, failed to be much impressed by the paper in which the masses of the electron and proton were calculated, they can read its arguments rather more cogently set out here.

The book deals with much more than the observational material which led to the concept of an expanding universe, this being dealt with in the first chapter. The next two chapters deal with the more theoretical reasons for expecting to find such a state of affairs, whilst the last chapter discusses, as has been implied above, the "argument of the 137" and the question "relatively to what is the universe expanding, and why do we not say instead that the contents are contracting?" The book is fascinating to read, its distinguished author displaying once again that lightness of touch, combined with sureness, for which he is well known.

The translation into French is on the whole happy, but the translator seems to have missed the point in his version of the lines which run in the original:

"He thought he saw electrons swift
Their charge and mass combine;
He looked again and saw it was
The cosmic sounding line.
The population then, said he,
Must be 10^{79}."

Perhaps we cannot expect him to be familiar with Lewis Carroll, but the facts that the second, fourth and sixth lines rhyme and particularly that the last one scans have been lost in the translation. J. H. A.

The Thermodynamics of Electrical Phenomena in Metals, by P. W. Bridgman. Pp. vi + 200. (London: Macmillan and Co., Ltd.) 16*s*.

The volume under review is essentially the substance of a number of papers which Prof. Bridgman has written during the last ten years on inter-relations of a thermodynamic character between various electrical properties of metals. The substance of these papers has now been consolidated into a coherent whole and published in book form.

The author points out that the development of the great fundamental concepts of physics is often a slow process, and this is particularly true of the theories of the electrical properties of matter. The historical development of the fundamental ideas spread over a long time; sixty years from Poisson to the formulation of the field equations for stationary bodies by Maxwell, to pick out two important landmarks.

The experience underlying the equations of Maxwell was a fairly exhaustive knowledge of electrical phenomena in empty space. The electrical properties of matter are, however, most complicated and many of them have been discovered since the crystallization of the concepts. Among the various properties of matter with which this book is specially concerned are the following: thermo-electric properties, including Peltier and Thomson thermal effects; the Volta effect; thermionic emission; photo-electric emission; cold discharges in intense fields; phenomena in crystals, including anisotropic resistance and various reversible heating effects; and various effects in the magnetic field, of which the Hall effect is the best known.

Prof. Bridgman has set himself the task of scrutinizing each of these effects to find whether it fitted into the scheme of fundamental concepts or whether modification was demanded. In the course of his enquiry he finds that there are relations between the various effects which are demanded by the broad general principles of thermodynamics. Apart from the intrinsic interest of these relations, many of which are new, they are of importance in directing the development of any detailed theory because it is necessary that the theory be capable of giving an account of them.

Another important conclusion arrived at is that the concepts in terms of which the attempt has been made to describe the electrical state inside a metal are not broad enough, but have to be amplified in a way which turns out not to be difficult.

E. G.

Operational Methods in Mathematical Physics, by H. JEFFREYS. Pp. vi + 117. (Second edition, Cambridge University Press.) 6*s*. 6*d*. net.

The first edition of this tract is probably better known to physicists than any other of the Cambridge Mathematical Tracts, and the second will be as useful. We may regret the disappearance of the discussion of Heaviside's generalized experimental series, but the sentiment is quite irrational. For the benefit of those who are not acquainted with the work, it may be said that the operational methods dealt with are those developed by Heaviside rather than those due to Boole. Heaviside did not know the reason why his methods worked, and the omission was repaired by Bromwich, who used the theory of contour integration, and later by Carson, who used integral equations. The present tract follows the path opened up by Bromwich, and is probably the only work not devoted solely to electrical networks which gives a connected account of the subject. Readers should not miss the first page, where the author, a Cambridge mathematician be it remembered, quotes Heaviside's remark, "Even Cambridge mathematicians deserve justice," nor the last page, where some penetrating remarks on differential equations have replaced the peroration of the first edition. If, in passing from the first to the last page, they follow the direct and not the retrograde route, they will be wise. J. H. A.

Cartesian Tensors, by H. JEFFREYS. Pp. vi + 92. (Cambridge University Press.) 5*s*. net.

Notation is a most important matter. Neither the D-ists nor those in their dot-age could have discovered Lagrange's equations in dynamics, because neither could express concisely the formula which we write $\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_r} - \frac{\partial T}{\partial q_r} = Q_r$. Again, we all like to name the point (x, y, z) as P so as to avoid frequent repetition of the group of symbols, but it is an advance to replace x, y and z by x_1, x_2 and x_3 and to abbreviate the three to x. The summation convention of tensor analysis is an analogous device. Dr Jeffreys has had the very useful idea of introducing the student to the tensor notion and notation in the particular case where the axes are always rectangular. Transformations are thus always linear and some of the difficulties of the general theory are consequently avoided, though its advantages are retained.

The applications of the theory to mechanics, including elasticity and hydrodynamics, are given, and the book is provided with that necessity for the true student, examples to be worked by the reader. It should do much to contribute to a wider knowledge of the subject, and is an excellent introduction to the more advanced treatises. J. H. A.

The Scientific Journal of the Royal College of Science, volume **4**. Pp. 172. (London: Edward Arnold and Co.) 7*s*. 6*d*.

This journal has reached its fourth annual volume and the editing committee is to be heartily congratulated on its success. To attempt to fulfil the double function of giving to the specialist a convenient and scholarly résumé of his own subject, and of assisting the layman in these matters to some comprehension of the work of the specialist, is to court disaster. But it can be done; and the chemist, say, will find here matter which will attract his interest as a specialist, and essays which will tell him something of recent advances in physics and in biology. Seven lectures in the chemistry section, six in the natural history, and four in the physics section, complete an excellent bill of fare. The Royal College of Science has got in first, and members of other and more slow-going London Colleges can only wish them heartily, and yet perhaps a little enviously, good fortune and good voyage in their endeavour. A. F.

Under the general heading "Actualités Scientifiques et Industrielles" we have received the monographs listed below. Each is written by an authority on his subject and the treatment is, in general, concise and clear. The publishers are Hermann et Cie., 6, Rue de la Sorbonne, Paris.

84. O. Scarpa. *Pile Metalliche.* 6 fr.

85. Prof. Dr M. Volmer. *Das elektrolytische Kristallwachstum.* 4 fr.

86. F. Bloch. *Les Électrons dans les métaux.* 5 fr.

87. A. F. Joffé. *Conductibilité électrique des isolants solides et des semi-conducteurs.* 10 fr.

88. Léon Brillouin. *Les Électrons dans les métaux du point de vue ondulatoire.* 9 fr.

89. Léon Brillouin. *Conductibilité électrique et thermique des métaux.* 18 fr.

90. J. Heyrovsky. *A Polarographic Study of the Electro-Kinetic Phenomena of Adsorption, Electro-Reduction and Overpotential, Displayed at the Dropping Mercury Cathode.* 12 fr.

91. René Audubert. *Phénomènes photoélectrochimiques action de la lumière sur le potentiel métal-solution.* 8 fr.

92. A. Gillet et N. Andrault de Langeron. *Les Colloïdes et la couche de passage.* 10 fr.

93. Paul Dutoit. *Sur le potentiel métal/solution.* 4 fr.

94. Georges Brooks. *Laque d'Indochine.* 18 fr.

97. Philipp Frank. *Théorie de la connaissance et physique moderne.* 10 fr.

98. P. Swings. *La Fluorescence des molécules diatomiques.* 10 fr.

99. P. Swings. *La Fluorescence des molécules diatomiques-phénomènes complexes.* 10 fr.

105. M. Prettre. *L'Inflammation et la combustion explosive en milieu gazeux.* 15 fr.

107. Jean Ullmo. *Les Idées d'Eddington.* 7 fr.

110. Jean-J. Trillat. *Les Preuves expérimentales de la mécanique ondulatoire.* 12 fr.

112. Sir Arthur Eddington. *Sur le problème du déterminisme.* 6 fr.

121. E. Darmois. *Un nouveau corps simple: le déuterium ou hydrogène lourd.* 7 fr.

122. G. Malfitano et M. Catoire. *Les Composés micellaires selon la notion de complexité croissante en chimie.* 9 fr.

130. Carl Benedicks. *Nouveaux résultats expérimentaux sur l'effet électro-thermique homogène.* 8 fr.

131. Lothar Nordheim. *Die Theorie der thermoelektrischen Effekte.* 6 fr.

132. Paul Langevin. *La Notion de corpuscules et d'atomes.* 12 fr.

140. Jean-Louis Destouches. *Les Principes de la méchanique générale.* 15 fr.

143. M. Georges Bouligand. *Relations d'incertitude en géométrie et en physique.* 9 fr.

152. Moritz Schlick. *Les Énoncés scientifiques et la réalité du monde extérieur.* 10 fr.

153. Louis Leprince-Ringuet. *Rayons cosmiques: aspect des phénomènes et méthodes expérimentales.* 15 fr.